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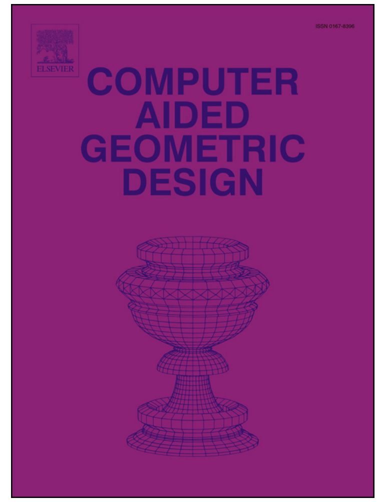

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Isogeometric analysis with strong multipatch C^1-coupling

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Abstract

C^1 continuity is desirable for solving 4th order partial differential equations such as those appearing in Kirchhoff-Love shell models [29] or Cahn-Hilliard phase field applications [15]. Isogeometric analysis provides a useful approach to obtaining approximations with high-smoothness. However, when working with complex geometric domains composed of multiple patches, it is a challenging task to achieve global continuity beyond C^0. In particular, enforcing C^1 continuity on certain domains can result in "C^1-locking" due to the extra constraints applied to the approximation space [9].

In this contribution, a general framework for coupling surfaces in space is presented as well as an approach to overcome C^1-locking by local degree elevation along the patch interfaces. This allows the modeling of solutions to 4th order PDEs on complex geometric surfaces, provided that the given patches have G^1 continuity. Numerical studies are conducted for problems involving linear elasticity, Kirchhoff-Love shells and Cahn-Hilliard equation.

1 Introduction

Isogeometric analysis, first introduced by [19], has been the subject of intensive study over the past decade, in part due to its promise of more closely integrating the analysis and design phases of product development. A distinct feature of this method is that the basis functions used for the discretization of the approximation space and the geometry description can have increased smoothness, up to C^{p-1}, where p is the polynomial degree of the basis. This is a significantly higher continuity order compared to the Lagrange polynomials commonly used in the finite element analysis, which are typically restricted to C^0. As a result, better efficiency in terms of degrees of freedom is observed when the solution is suitably smooth, and 4th (as well as higher) order partial differential equations (PDEs) can be approximated using standard discretization methods.

Unfortunately, on complex geometries where multiple parameter spaces (patches) are joined together to describe the physical domain, there is typically a loss of continuity which occurs at the patch boundaries. This decrease of smoothness is dictated by the geometry description, where C^0 parameterizations are normally used to deal with kinks and corners in the domain. Multiple patches also need to be used to describe complex domains such as those with inclusions, since a single patch is limited by its underlying tensor product structure to describing relatively simple shapes.

Several methods have been proposed in engineering literature to deal with the decreased smoothness at the patch interfaces, such as the bending strip method or Kirchhoff-Love shells [28], which employs a fictitious material along the patch interface to approximately satisfy the kinematic constraints. This method was also used in [38] to couple trimmed NURBS patches. Other approaches include mortar methods [5, 12, 17], where adjacent patches are related by a master-slave relation and Lagrange multipliers are used to enforce continuity in a variational sense. The most accurate and stable development to date seems to be the Nitsche's method [16, 1, 35, 37], where a mesh-dependent penalty (stabilization) term is used. These methods allow the coupling of non-conforming patches, however, they may lead to semi-definite saddle point problems in the case of mortar or Lagrange multiplier method. Nitsche's method also requires the determination of the stabilization parameters, which increases the overall cost.

A different way of ensuring that the desired continuity requirements are satisfied is to construct a basis for the approximation space, where the basis functions themselves have the required smoothness. Piecewise polynomial bases with C^1 continuity have been derived using symbolic algebra for planar bilinear patches in [25, 21]. A similar approach was used to obtain C^2 bases for planar domains in [24, 23]. These functions span a subspace of the C^0 or C^{-1} spline spaces defined on each patch individually and therefore result in a reduction in the number of degrees of freedom compared to the full (with less regularity) approximation space. A possible issue with this method was noted in [9], where it was shown that the over-constraining of a piecewise polynomial subspace of a given degree over certain geometries could result in a loss of approximation properties (C^1 locking). This has led to the study of analysis-suitable (AS) parameterizations [22], which include bilinear and bilinear-like planar mappings.

A related problem is the construction of smooth spline spaces over unstructured meshes, such as those obtained using T-Splines or even standard finite element mesh generators. Particular attention has been devoted to the parametrization around extraordinary vertices (interior vertices on quadrilateral meshes which have valence different than 4), where the desired smoothness properties have to be imposed through additional constraints. This leads to the "capping problem", where a ring of elements around the extraordinary vertex needs to be adjusted, for which various techniques have been proposed in [34, 41, 26]. In [40], smooth polar splines have been used at the extraordinary vertex, while a mathematical analysis of the dimension and basis construction on arbitrary topologies is presented in [32]. A construction using Hermite splines with optimal approximation properties is described in [44], while a more general method for smooth approximations over unstructured meshes is given in [4].

Another common approach for constructing smooth surfaces is through the use of subdivision surfaces, for which applications, in particular to thin-shell analysis have seen increased interest, see [6, 43]. However, while subdivision surfaces provide a convenient way to construct smooth surfaces, they are not yet widely used for engineering applications. Alternatively, T-Splines have been used more extensively for modeling surface and plane geometries [3, 11], using both unstructured and hierarchical meshes. Both the T-Spline and subdivision surface meshes require particular attention near the extraordinary vertices where the smoothness or approximation properties may be reduced when used in the analysis. Moreover, several approaches have been developed for discretizing the interior of the domain from a boundary triangulation or CAD surface geometry using T-Splines [14, 46], or Bézier tetrahedra [13, 45].

In this work, we follow a constructive approach for the approximation space, while leaving the geometric parametrization unchanged. We consider general geometries which are not limited to planar or bilinear mappings. A suitable basis is given in terms of Bézier-Bernstein polynomials, whose coefficients are numerically computed based on the given geometry. The problem