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Reconstruction of helices from their orthogonal projection ☆

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ABSTRACT

We describe a method for modeling helices from planar curves. Given a polygonal curve in the (x, y) plane, the method computes a helix such that its orthogonal projection onto the (x, y) plane fits the polygonal curve. The helix curve is first sampled and the transformation matrix that best aligns points of the sampled helix to those of the polygonal curve is calculated. This transformation matrix is then used to estimate the parameters of the helix whose projection fits the polygonal curve.

1. Introduction

Helices are three-dimensional curves whose curvature and torsion are constant. These curves are very common in nature and among human-made objects. Examples of objects having the shape of helix are hair curls, coiled phone cords, spring coils, tendrils of climbing plants, etc.

In this paper, we focus on the modeling of circular helices from their orthogonal projection. Our algorithm takes as input a polygonal curve in the plane $z = 0$ and generates a helix segment such that its orthogonal projection on the plane $z = 0$ best approximates the input polygonal curve. Our contribution is a method which is much more robust with respect to noise in the input data than the previous ones.

In this work, we aim at reconstructing a single helix from its orthogonal projection. There are many cases for which such method would be useful. The first application is related to computer vision; our algorithm could be used to reconstruct circular helices from an image taken with a camera. There are many examples of objects that may have the shape of circular helix: a photo of the architectural structure of a building, the image of the DNA structure taken with an electron microscope, etc. Another application of our algorithm could be for the reconstruction of helices from scan digitized images of engineering drawings. Engineering drawings are usually composed of primitives such as segments of lines, circles, ellipses and orthogonally projected helices. The digitized image would be first converted into a vector image using existing approach (Hilaire and Tombre, 2006). The helices could then be reconstructed from the curves that compose the vector image.

2. Related work

In this section, we review existing methods that have been developed for the modeling of circular helices. We also describe some of the methods in the domain of sketch-based modeling that could be used for the modeling of 3D curves.

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2.1. Sketch-based modeling of wireframe models

One of the first approaches in sketch-based modeling is to let the user to draw the shape in wireframe. The input sketch is composed of a set of straight lines segments connected to each other and located in a plane; the goal is to compute the third coordinates of the segment extremities to create a 3D shape such that its projection matches the input sketch. The reconstruction is often solved with an optimization whose unknown variables are the third coordinates of the segment extremities. Different objective functions have been proposed such as minimizing the entropy of the angle distribution (Shoji et al., 2001) or minimizing the standard deviation of the segment lengths (Brown and Wang, 1996). The main drawback of these methods is that they cannot be used to reconstruct curvilinear shapes; they assume that the shape to reconstruct is rectilinear.

Cordier et al. (2013) have proposed a method to reconstruct curvilinear 3D shapes from a set of planar curves. The main limitation is that the reconstructed shape is assumed to be mirror-symmetric; their method cannot be extended to process non-symmetric shapes.

2.2. Approximating curves with piecewise helix curves

Several researchers have worked on the problem of fitting helices to 3-dimensional polygonal curves or 3-dimensional point clouds (Christopher et al., 1996; Nievergelt, 1997; Enkhbayar et al., 2008; Piuze et al., 2011; Goriely et al., 2009; Ghosh, 2010; Derouet-Jourdan et al., 2013). These algorithms are not directly applicable to the reconstruction of helices from orthogonal projection.

2.3. Reconstruction of helices from their orthogonal projection

Several researchers have worked on the problem of finding a helix whose orthogonal projection matches a polygonal curve provided by a user. In the work by Wither et al. (2007), the axis of the reconstructed helix is assumed to be parallel to the projection plane. Our method is able to handle the reconstruction of helices with arbitrary orientation.

Marchal et al. (2009) proposed a method to reconstruct trochoids using the derivative of the input curve with respect to its arc-length. Trochoids are special case of orthogonal projection of helices. One limitation of their method is that it is restricted to the reconstruction of x-axis aligned trochoids.

The closest work to ours is the one proposed by Chérin et al. (2014). Similarly to our method, their approach is aimed at finding a helix whose orthogonal projection matches a planar polygonal curve. Compared to their method, ours is much more robust with respect to noises in the input polygonal curve. We provide a detailed comparison of the two methods in the result section to support this claim.

3. Overview

Our method takes as input a 2D polygonal curve C in the (x, y) plane and generates a helix segment such that its orthogonal projection onto the (x, y) plane fits the polygonal curve. Note that the problem of fitting an orthogonally-projected helix segment to a polygonal curve is an underdetermined problem and therefore may not have a unique solution.

Instead of computing the fitting of the projected helix with the polygonal curve C, we simplify the problem by sampling the helix and compute the alignment of the points of the sampled helix with those of the polygonal curve. The problem comes down to estimating the transformation matrix that best aligns the points of the sampled helix with those of the polygonal curve. Using this optimal transformation matrix, we then estimate the parameters of the helix whose projection best fits the polygonal curve. This helix fitting method is explained in Section 4.

One important assumption of using this alignment method is that there is a one-to-one correspondence between the points of the sampled helix and those of the polygonal curve C. This implies that the helix and the polygonal curve C must be sampled such that this correspondence is valid. In section 5, we present an adaptive sampling method that satisfies this property. In section 6, we describe how this adaptive sampling is used in combination with the helix fitting.

The main contribution of the paper is to propose a new method for the fitting of an orthogonally-projected helix segment to a polygonal curve. Compared to the state-of-the-art methods, this new method is much more robust with respect to noise in the input polygonal curve. In section 7, we provide a detailed comparison of our method with the previous one.

4. Computing the helix parameters and the fitting error

Let C be a polygonal curve in the (x, y) plane. C is composed of n points $\{p_1, p_2, \ldots, p_i \ldots, p_n\}$; each point p_i has coordinates $(x_{P,i}, y_{P,i})$. Let $H(t)$ be the parametric equation of a helix:

$$H(t) = \begin{bmatrix} r\cos(t) \\ pt \\ r\sin(t) \end{bmatrix}$$