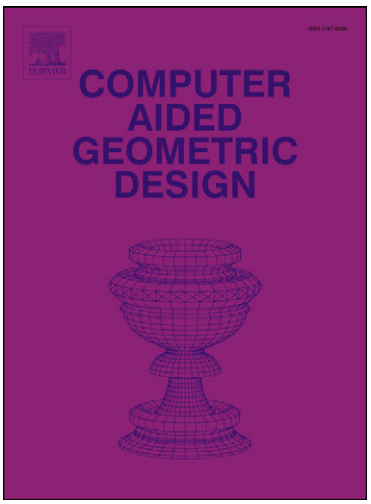

Classical curve theory in normed planes

Horst Martini, Senlin Wu

PII: S0167-8396(14)00045-4
DOI: 10.1016/j.cagd.2014.03.003
Reference: COMAID 1421

To appear in: *Computer Aided Geometric Design*

Please cite this article in press as: Martini, H., Wu, S. Classical curve theory in normed planes. *Computer Aided Geometric Design* (2014), http://dx.doi.org/10.1016/j.cagd.2014.03.003

Classical curve theory in normed planes

Horst Martini Senlin Wu

May 20, 2014

Abstract

The classical theory of individual classes of planar curves is a well-known field between Elementary, Differential, and Algebraic Geometry. With the present expository paper we want to point out the fact that an extension of this field to two-dimensional real Banach spaces, also called Minkowski planes, is still missing. We want to show that until now only a few topics from this natural and rich geometric field were extended to Minkowski planes, and that, moreover, even in these directions only partial results exist. We present these known results, give open problems and show up possible directions of future research. It is our goal to verify that classical curve theory in Minkowski planes can be nicely developed to become a very wide and interesting research subject in the spirit of different modern fields, like Differential Geometry, Functional Analysis, Computational Geometry and related directions.

MSC(2000): 14H45, 14H50, 51L10, 51M04, 51N20, 52A10, 52A40, 53A04

Keywords: antinorm, angular bisector, arc-length parametrization, Birkhoff orthogonality, bisectors, Cassini curves, circles, Clifford's chain of theorems, conics, curves of constant width, curves of constant area-halving distance, equiframed curves, geometric dilation, halving distance, isodiametric problem, isoperimetric problem, midpoint curve, minimum chords, Minkowski geometry, Miquel's theorem, multifocal Cassini curves, multifocal ellipses, normed planes, polyellipses, Radon curves, Zindler curves

1 Introduction and basic notions

The classical theory of *special types of planar curves* (like, e.g., Cassini curves, cycloids, conchoidal curves, sinusoidal spirals, etc.) is an inspiring and beautiful field "between" Elementary, Differential, and Algebraic Geometry, with many fascinating applications in Physics, Astronomy, Classical Mechanics, Engineering, and various further disciplines. Wider representations on concrete curve classes in the Euclidean plane can, unfortunately, only be found in older books like, e.g., [54], [95], and [32]. These books contain a great deal of historical information on and properties and applications of individual classes of curves, which is (almost) not to be found in the standard works on curves published in English language. Most of these stress the general theory and deal with special curves only by way of examples: the exhaustive treatise by Coolidge [18], to mention at least one example, contains almost no figures of plane curves. More recent books with the title "Algebraic Curves" usually have a different outlook altogether. They are intended as introductions to modern Algebraic Geometry, so that a reader who wants to find something about Clairaut's multiplicatrices or cissoidal curves, or on the interrelation between different formulae of curves and their