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Overconstrained mechanisms based on planar four-bar-mechanisms

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ABSTRACT

We study a particular class of planar four-bar mechanisms *FBM*(Q) which are based on a given quadrilateral (quad) $Q = \mathbf{a}_0\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3$. The self-motion of *FBM*(Q) consists of two different parts – one is the motion of an anti-parallelogram whilst the other one is a pure translation with circular paths. We will refer to this translatoric part in this paper only and demonstrate that this translatoric self-motion has the following property: At any moment the positions of the corresponding four coupler points form quads homothetic to Q. This property can be used to define spatial one-parametric motions of an extruded version of the four-bar mechanism which again generate quads of coupler points homothetic to Q. As the next step we take an arbitrary "saturated chain" of quads in space (each vertex shares a vertex with another quad of the set) and define the corresponding one-parametric spatial motions. Then all can be parametrized by the same parameter t. We will show that these partial motions can be interlinked by spherical $2R$-joints without locking the one-parametric self-motion. This way the construction delivers a series of new (overconstrained) mechanisms which generalize results on so-called "Fulleroid" linkages. An example based on four quads in space (in planes of a tetrahedron) is worked out in detail.

1. Introduction

The aim of the paper is to generalize the construction of highly overconstrained mechanisms consisting of rigid bodies linked by $1R$- or spherical $2R$-joints.[1] Based on the famous Heureka mechanism and its generalizations (see Stachel, 1991, 1992 and Röschel, 1995, 1996a, 2001) the so-called Fulleroid mechanisms have been studied in the last years by Kiper et al. (2008), Kiper (2008), Wohlhart (1997, 2001, 2007) and Röschel (2010, 2012).

The considerations are based on an observation on special planar four-bar mechanisms: In Section 2 we will define special four-bar mechanisms based on quadrilaterals (quads) and work out an interesting property. In Sections 3 and 4 the planar results are embedded into space and yield further generalizations of the so-called Fulleroid linkages. In Section 5 the results are worked out in detail for a special example based on quads in the faces of a tetrahedron. Section 6 is devoted to the study of bifurcations of the mechanism's self-motions.

[1] A $2R$-joint is called "spherical" if its two rotary axes intersect in a point.

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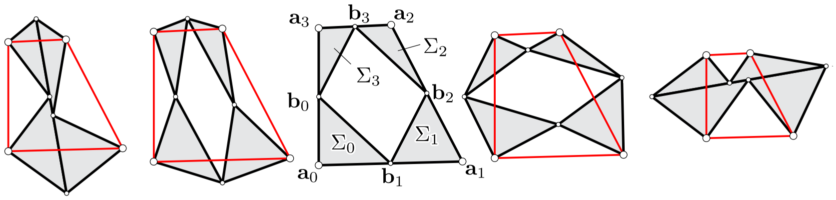

Fig. 1. The quadrilateral $Q = \mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ and the corresponding parallel four-bar mechanism at several positions of the parallel self-motion.

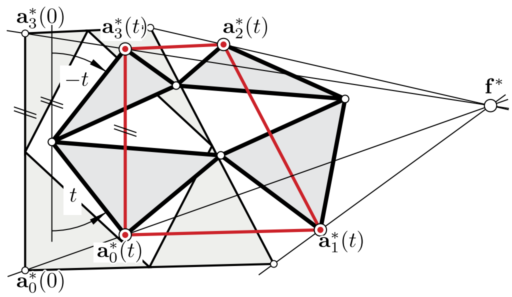

Fig. 2. Two positions of $\zeta(Q, t)$ and the center of homothety.

2. Quadrilaterals and an observation in planar kinematics

Let $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ be the vertices of a quadrilateral ("quad") Q in the Euclidean plane. The midpoints of the pairs of points $\mathbf{a}_{i-1}\mathbf{a}_i$ (with indices modulo 4) are the points $\mathbf{b}_i := (\mathbf{a}_{i-1} + \mathbf{a}_i)/2$ – see Fig. 1.[2] Due to our construction the midpoints $\mathbf{b}_0, \ldots, \mathbf{b}_3$ form a parallelogram with sides parallel to the diagonals of the quad.

The eight points $\mathbf{a}_i$ and $\mathbf{b}_i$ define four triangles $\mathbf{b}_i\mathbf{a}_i\mathbf{b}_{i+1}$ (shaded in Fig. 1) which can be viewed as rigid bodies Σ_i linked via rotary joints in the points $\mathbf{b}_i$ ($i = 0, \ldots, 3$). The corresponding planar mechanism is a parallel four-bar mechanism (see Wunderlich, 1970) which we call *parallel (or special) four-bar mechanism FBM*(Q) *based on the quad* Q. Basically, the self-motion of *FBM*(Q) consists of two different parts – one is the motion of an anti-parallelogram whilst the other one is a pure translation with circular paths. We will refer to this translatoric part in this paper only and call it $\zeta(Q, t)$. Under $\zeta(Q, t)$ the points $\mathbf{b}_i$ form parallelograms at any moment $t \in I \subset \mathbb{R}$ of the motion.

We will describe the self-motion $\zeta(Q, t)$ as the composition of partial one-parametric motions $\zeta_i(t)$ of the systems Σ_i with respect to a fixed *world-coordinate frame* Σ^*. We will denote the rotational angles of ζ_i by ϕ_i ($i = 0, \ldots, 3$). As opposite bars of a parallel four-bar mechanism rotate with the same angular velocity, we can take

$$\phi_0(t) = \phi_2(t) := t \in I := [0, 2\pi] \subset \mathbb{R}. \tag{1}$$

For ζ_1 and ζ_3 we will take $\phi_1(t) = \phi_3(t) := -t$. The rotations $\rho_i(t)$ of Σ_i about the center $\mathbf{a}_i$ then are parametrized by

$$\begin{aligned} \rho_i(t) : \Sigma_i &\to \Sigma^* \\ \mathbf{p} &\to \rho_i(\mathbf{p}, t) := \mathbf{a}_i + R\big(\phi_i(t)\big)(\mathbf{p} - \mathbf{a}_i) \end{aligned} \tag{2}$$

with

$$R\big(\phi_i(t)\big) = \begin{pmatrix} \cos t & (-1)^{i+1} \sin t \\ (-1)^i \sin t & \cos t \end{pmatrix}. \tag{3}$$

They determine $\zeta_i(Q, t)$ up to some further translational parts. Fig. 1 displays such poses of the four-bar mechanism. The segments $\mathbf{a}_0\mathbf{b}_0$ and $\mathbf{a}_3\mathbf{b}_0$ are rotated by $\pm t$ – see Fig. 2. At $t = 0$ we have $\mathbf{a}_i^*(0) := \rho_i(\mathbf{a}_i, 0) = \mathbf{a}_i$ ($i = 0, \ldots, 3$). The straight lines through the initial points $\mathbf{a}_3^*(0)\mathbf{a}_0^*(0)$ and the corresponding positions $\mathbf{a}_3^*(t)\mathbf{a}_0^*(t)$ are parallel for all $t \in [0, 2\pi]$. Their distances are scaled by the factor $\cos t$. As the same holds for the other pairs of coupler points and their positions the quads formed by the coupler points are similar to the initial one. At any moment there exists a corresponding center of homothety.

Now we will add translational parts of the partial motions such that this center of homothety is kept in its place. We choose a point $\mathbf{f}^*$ in Σ^* and define the partial motions via

[2] In the three-dimensional Euclidean space (or the plane) we use coordinates with respect to a given Cartesian coordinate frame.

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