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As-rigid-as-possible solid simulation with oriented particles

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We propose a new as-rigid-as-possible approach to the real-time simulation of physics-based deformable models for interactive applications such as computer games. The key observation is that the efficacy of an embedded oriented particle representation and the stability of a variational implicit formulation of the projective dynamics are complementary to each other. We reformulate the variational implicit formulation to deal with an embedded graph of oriented particles. Our new formulation is extremely stable, and our alternating local/global optimization solver is both easy to implement and computationally efficient. Our method can deal with one-dimensional (cable and rod), two-dimensional (shell), and three-dimensional (solid) models in a uniform manner. Experimental results demonstrate that hundreds of deformable models with an extremely large number of polygons can be simulated robustly in real time using thousands of particles.

1. Introduction

Physics-based simulation is becoming one of the most important techniques for interactive applications such as games. In this paper, we consider physics-based deformation, which has been studied extensively in computer graphics. However, it is still challenging to robustly produce visually convincing and stable deformation in real time. In real-time applications, the robustness and computational efficiency of deformation simulations are often more important than their accuracy.

Recently, several simulation techniques have been developed to compute positions directly instead of integrating velocities or accelerations to achieve better stability and efficiency. Position-based dynamics (PBD) [1] deals directly with mesh vertices, while meshless deformation [2] utilizes shape matching to compute the optimal rotation and translation for mesh vertices. These position-based methods are stable regardless of the time step size, and thus the deformation can be simulated very efficiently with large time steps.

Two remarkable approaches aim to further improve the PBD method. The first approach employs a simplified structure with a small number of oriented particles to simulate the complex geometry of meshes in a more efficient and robust manner [3]. The second approach called projective dynamics [4] reformulates the implicit time integration of deformation dynamics as energy minimization in a variational form. In PBD, the material stiffness of a deformable solid is tightly coupled with the convergence of the solver. The formulation of variational implicit integration is similar to PBD, but the projective dynamics method is advantageous because the material stiffness can be specified independently of the solution methods.

We found that these two approaches are complementary to each other and they can be combined to take advantage of their benefits. The key challenge is to reformulate the deformation energy and momentum potential energy to deal with an embedded graph of oriented particles. Our new formulation yields a compact formula via clever manipulation of the integral energies and it is extremely stable. Furthermore, our alternating local/global optimization solver is easy to implement and very efficient for simulating complex deformable models in real time. The deformable models can be manipulated interactively and the collision between deformable models can be handled efficiently. Our method can deal with one-dimensional (cable and rod), two-dimensional (shell), and three-dimensional (solid) models in a uniform manner.

2. Related work

Physics-based simulations of deformable bodies have been researched for decades in computer graphics since the pioneering work by Terzopoulos et al. [5] and many methods have been developed to produce accurate simulations of various types of deformable objects [6]. However, in real-time applications, the robustness and computational efficiency of deformation simulations

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are often more important than their accuracy, and these requirements are still demanding.

Recently, a number of methods have been proposed for robust real-time simulations by evolving the positions of the particles using the initial and predicted positions, before updating the velocities based on the positions. Meshless deformation based on shape matching computes the optimal rotation and translation from the initial positions to the predicted positions to deform the mesh vertices [2]. This method is robust regardless of the time step size, and thus it is suitable for real-time applications. However, shape matching with a single transformation is restricted to modest deformation. The extension of this method to a lattice applies shape matching to each set of overlapping lattice points with a fast summation method to generate large deformation [7]. The fast summation approach has been applied to the simulation of hair with chain shape matching [8], as well as being generalized to an irregular structure on a surface mesh with volume preservation [9], and extended to multi-resolution approaches to enhance the convergence [10,11]. Shape matching with oriented particles is an efficient and robust method for simulating the complex dynamic deformation of one-, two-, and three-dimensional deformable bodies even with a small number of particles in a single framework [3]. A deformable body is approximated by ellipsoidal particles and shape matching is applied iteratively to each shape matching group comprising a particle and its one-ring neighbors. Our method employs this oriented particle representation so hundreds of deformable models with an extremely large number of polygons can be approximated using thousands of particles and simulated robustly in real time, as illustrated in Fig. 7.

Shape matching can be interpreted as a geometric constraint in PBD [1]. PBD employs an iterative mass-weighted projection of constraints in a Gauss–Seidel solver. However, the stiffness is highly dependent on the time step size and the iteration count, which is also problematic in shape matching approaches. Nevertheless, PBD is popular for the real-time simulation of deformable models because of its simplicity and robustness. It has been extended to the simulation of fluids [12], rigid bodies [13], elastic rods [14,15], and volumetric materials [16], as well as all of them in a unified manner [17]. A comprehensive survey of PBD methods was provided by Bender et al. [18]. Recently, PBD was extended to address the stiffness independently of the time step size and the iteration count based on a compliant constraint formulation [19]. However, the extended PBD still employs Gauss–Seidel iterations, and thus its convergence is slower than that of projective dynamics [4,20] where the global solver benefits from the pre-factored system matrix. Moreover, its application to oriented particles has not been addressed previously.

Recently, a number of fast and parallel techniques have been proposed by formulating implicit Euler integration as energy minimization [21,22] and by employing an alternating local/global optimization solver. Liu et al. [20] proposed a local/global solver for mass-spring systems, where the local solver deals with nonlinear terms for direction and the global solver handles stretching. This idea was generalized to other constraints in projective dynamics [4]. A Chebyshev semi-iterative approach was also proposed, which combines the results obtained from previous iterations to achieve better convergence in projective dynamics and PBD [23]. Narain et al. [24] employed the alternating direction method of multipliers (ADMM) for implicit time integration and showed that projective dynamics is a special case of ADMM. Their method also allows nonlinear constitutive models and hard constraints. In addition, Liu et al. [25] interpreted projective dynamics as quasi-Newton optimization and applied the L-BFGS method to accelerate convergence. In these techniques, the stiffness is largely independent of the iteration count and the solution becomes more accurate as the number of iterations increases. Our method is also based on implicit Euler integration formulated as energy minimization, and thus it differs from the original oriented particles approach where the stiffness is highly dependent on the time step size and the iteration count.

In as-rigid-as-possible (ARAP) surface modeling [26], a block coordinate descent method is employed to iteratively minimize the shape deformation energy in alternating local/global optimization steps. This method introduces a local rotation at each vertex to define an ARAP deformation energy by using the squared distances between the locally rotated positions of its neighbors and the actual deformed positions. The auxiliary rotations and the positions of the vertices should be optimized. An optimal rotation at a vertex is computed in parallel with shape matching of its neighboring vertices, which requires the polar decomposition of a shape matching matrix. The optimal positions are computed efficiently by solving a linear system, which depends only on the initial mesh, so it can be pre-factored with a sparse Cholesky decomposition.

Embedded deformation for shape manipulation [27] introduces a deformation graph of nodes corresponding to rigid transformations that deform nearby space, and then defines an ARAP deformation energy over the deformation graph. In contrast to ARAP surface modeling, this approach can deal with a wide range of shape representations, such as meshes, polygon soups, mesh animations, and animated particle systems. However, the node has no volume, and thus it cannot deal with the deformation of a one-dimensional structure robustly, unlike the oriented particles approach and our proposed method. In addition, the iterative Newton–Gauss method employed for nonlinear optimization requires more time than the iterative local/global optimization method when only seeking a plausible solution in a small number of iterations.

3. Method

In this paper, we introduce a deformation graph $\mathcal{G}$ comprising oriented particles, which approximate a flexible body and deform nearby spaces robustly even with a small number of particles. The positions and orientations of the oriented particles are computed stably by solving an energy minimization problem formulated as implicit Euler integration in a variational form. The total energy $E(\mathcal{G})$ comprises the momentum potential energy $E^k(\mathcal{G})$, ARAP deformation energy $E^e(\mathcal{G})$, and direct manipulation constraint energy $E^c(\mathcal{G})$:

$$E(\mathcal{G}) = E^k(\mathcal{G}) + E^e(\mathcal{G}) + E^c(\mathcal{G}). \quad (1)$$

We develop an iterative local/global optimization solver to seek a plausible solution in an efficient and robust manner. The final deformed vertices are obtained by linear blend skinning of the rigid transformations stored in the oriented particles, which can be implemented with GPU skinning.

Our main contribution lies in developing the ARAP deformation energy over a deformation graph comprising oriented particles. Thus, we first explain the deformation graph and the corresponding space deformation in Section 3.1, before deriving the ARAP deformation energy in Section 3.2. We explain the alternating local/global optimization solver before considering other energies to make the explanation clearer. The momentum potential energy and direct manipulation constraint energy are described in Sections 3.3 and 3.4, respectively.

3.1. Deformation graph

Suppose that the jth ellipsoidal particle $\mathcal{E}_j$ is transformed from the rest position $\bar{\mathbf{x}}_j$ and orientation $\bar{\mathbf{E}}_j$ to the current position $\mathbf{x}_j$ and orientation $\mathbf{E}_j = [\mathbf{e}_j^a|\mathbf{e}_j^b|\mathbf{e}_j^c]$, where $\mathbf{e}_j^a$, $\mathbf{e}_j^b$, $\mathbf{e}_j^c$ are the current unit axes of $\mathcal{E}_j$. We denote the set of particles directly connected to