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A hierarchical factorization method for efficient radiosity calculations

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ABSTRACT

The radiosity problem can be expressed as a linear system, where light interactions between patches of the scene are considered. Its resolution has been one of the main subjects in Computer Graphics, which has led to the development of methods focused on different goals. For instance, in inverse lighting problems, it is convenient to solve the radiosity equation thousands of times for static geometries. Also, this calculation needs to consider many (or infinite) light bounces to achieve accurate global illumination results. Several methods have been developed to solve the linear system by finding approximations or other representations of the radiosity matrix, because the full storage of this matrix is memory demanding. Some examples are hierarchical radiosity, progressive refinement approaches, or wavelet radiosity, which may become slow for many bounces. Recently, new direct methods have been developed based on matrix factorization.

This paper introduces a novel and efficient error-bounded factorization method based on the use of multiple singular value decompositions and the Z-order curve to sort the patches of the model. This technique accelerates the factorization of in-core matrices, and allows to work with out-of-core matrices passing only one time over them. Using this method, the inverse of the radiosity matrix can be efficiently approximated, reducing the memory and time resources needed to compute radiosity with infinite bounces. In the experimental analysis, the presented method is applied to scenes up to 163 K patches. After a precomputation stage, it is used to solve the radiosity problem for fixed geometries at interactive times.

1. Introduction

Radiosity is a method for global illumination (GI) calculation on scenes with Lambertian surfaces. This method has been the subject of study in several areas such as computer animation, architectural design and heat transfer [1]. To implement this technique, iterative methods have been studied and used [2,3], failing to compute solutions at real-time with many bounces. Other GI methods were proposed and developed, such as *instant radiosity* [4], *precomputed radiance transfer* [5], or *GPU-based global illumination* [6]. These techniques allow modeling many light effects, taking advantage of the new hardware architectures. Nevertheless, they also fail to provide real-time solutions when simulating many bounces of light. Recently, new techniques have been developed to solve the real-time infinite bounce radiosity problem, for a static geometry and dynamic emission [7,8].

Usually, due to spatial coherence, the form factors matrix [9] (**F**, see Table 1) has a low numerical rank. This fact enables the application of factorization techniques to compute low rank approximations of **F** that can be stored in main memory. Matrix factorization [10] is applied in several subjects related to mathematics and computer sciences. It is useful when matrices with low numerical rank are present in the problem. Nevertheless, it is not always simple to compute the factorization due to memory and execution time reasons. Hence, it is important to continue developing techniques to exploit the different characteristics offered by every specific problem.

In this work we present a hierarchical factorization (HF) technique for radiosity calculations. This technique implements a divide-and-conquer strategy based on the calculation of multiple singular value decomposition (SVD). In addition, the sorting of rows/columns of the matrix **RF** by similarity is exploited to speed-up the technique. For this purpose, we use the Z-order curve [11] over the patches of the scene, because each patch is associated to a row/column in the matrix. This method is intended to accelerate the factorization of in-core and out-of-core **RF** matrices, requiring just one pass.

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Table 1
Symbol notation and meaning.

Symbol	Definition
HF	Hierarchical factorization method
n	Number of patches
k, r	Matrix rank
σ_{max}	Largest singular value ($\sigma_{max}=\sigma_1$)
σ_i	i^{th} singular value
ε	Expected error
ε_p	Expected error of parent node
ε_{ch}	Expected error of child node
μ, σ	Mean and standard deviation
u, v	Left and right singular vectors
v_{max}	Right singular vector associated to σ_{max}
B	Radiosity vector
$\tilde{B}$	Approximation to the radiosity vector
E	Emission vector
$\mathbf{I}$	$n \times n$ identity matrix
$\mathbf{I}_k$	$k \times k$ identity matrix
$\mathbf{R}$	Diag. matrix with reflectivity indexes
$\mathbf{F}$	Form factors matrix
$\mathbf{RF}$	Matrix result of the product $\mathbf{R} \cdot \mathbf{F}$
$(\mathbf{I}-\mathbf{RF})$	Radiosity matrix
$\mathbf{U}_k$, $\mathbf{V}_k$	$n \times k$ matrices, $\mathbf{U}_k\mathbf{V}_k^T \approx \mathbf{RF}$
$\mathbf{Q}_k$	$\mathbf{U}_k\mathbf{D}_k$
$\mathbf{D}_k$	Diagonal matrix with $\sigma_1 \ldots \sigma_k$
$(\mathbf{A}_1 \mid \mathbf{A}_2)$	Matrix partition in two column blocks

The rest of the paper is organized as follows. Section 2 introduces some related work, while Section 3 describes the proposed algorithm and studies its error estimation. The experimental analysis is reported in Section 4, including studies on the performance of the algorithm and a comparison with other factorization techniques. Finally, in Section 5 the research is summarized and some lines of future work are defined.

2. Related work

This section introduces some previous work related to our proposal, which includes the radiosity problem and some state-of-the-art factorization techniques.

2.1. The radiosity method

The radiosity equation (Eq. (1)) computes the illumination of a scene by solving a linear system [12].

$$(\mathbf{I}-\mathbf{RF})B = E \tag{1}$$

In this equation, all matrices have $n \times n$ dimension, where n is the number of patches. $\mathbf{I}$ is the identity matrix, $\mathbf{R}$ is a diagonal matrix containing the reflectivity of each patch and $\mathbf{F}$ is the form factors matrix, where each cell (i,j) indicates the fraction of light reflected or emitted by patch i that arrives into patch j. Each row of the $\mathbf{F}$ matrix is computed based on the scene view from the corresponding patch. When close patches have similar views (there is spatial coherence in the scene), the $\mathbf{F}$ matrix has many similar rows/columns. Finally, B and E are vectors containing the radiosity value (W/m^2) of each patch and its emission, respectively.

2.2. Factorization of the **RF** matrix

There is a clear relation between the spatial coherence of a scene and the low numerical rank of the matrix $\mathbf{RF}$. References about low-rank properties of radiosity and radiance matrices can be found in Baranoski et al. [13], Ashdown [14], Hasan et al. [15], and Fernández [7]. This property allows us to approximate $\mathbf{RF}$ by the product of two matrices $\mathbf{U}_k\mathbf{V}_k^T$, both with dimension $n \times k$ ($n \gg k$), without losing relevant information.

The memory required to store $\mathbf{U}_k$ and $\mathbf{V}_k^T$ is $O(nk)$, while for $\mathbf{RF}$ it is $O(n^2)$. When $n \gg k$, the memory savings can be significant, even allowing to store them in system memory for large scenes.

If $\mathbf{RF}$ is replaced by its approximation $\mathbf{U}_k\mathbf{V}_k^T$ in Eq. (1), Eq. (2) is obtained, where the radiosity B is now transformed into its approximation $\tilde{B}$.

$$\left(\mathbf{I}-\mathbf{U}_k\mathbf{V}_k^T\right)\tilde{B} = E \tag{2}$$

Using the Sherman–Morrison–Woodbury formula [10], the matrix $\left(\mathbf{I}-\mathbf{U}_k\mathbf{V}_k^T\right)$ is inverted:

$$\tilde{B} = E+\mathbf{U}_k\left(\left(\mathbf{I}_k-\mathbf{V}_k^T\mathbf{U}_k\right)^{-1}\left(\mathbf{V}_k^T E\right)\right) \tag{3}$$

Hence, $O(nk^2)$ operations and $O(nk)$ memory are required to find $\tilde{B}$ using Eq. (3).

Eq. (3) can be transformed as follows:

$$\tilde{B} = E-\mathbf{Y}_k\left(\mathbf{V}_k^T E\right) \tag{4}$$

where $\mathbf{Y}_k = -\mathbf{U}_k\left(\mathbf{I}_k-\mathbf{V}_k^T\mathbf{U}_k\right)^{-1}$

$\mathbf{Y}_k$ is an $n \times k$ matrix. When the geometry and the reflectivity of the scene are static, $\mathbf{U}_k$, $\mathbf{V}_k$, and $\mathbf{Y}_k$ are computed only once in a pre-computation stage. After $\mathbf{Y}_k$ is found, the calculation of $\tilde{B}$ has complexity $O(nk)$. This result is useful when the radiosity equation needs to be solved many times, and E is the only element modified in Eq. (2).

One example of a factorization technique applied to radiosity problems is the Low Rank Radiosity (LRR) method [8]. This method allows us to factorize the $\mathbf{RF}$ matrix into one full matrix and one sparse matrix. This technique is relatively fast and can be computed by passing just one time over the whole matrix. The main problem of this technique is that two different scene meshes are needed, with two different granularity levels (*coarse* and *fine meshes*). The number of patches of these meshes define the dimensions k and n of the factorization matrices. For many scenes, it is not possible to define a coarse mesh with a small enough number of patches, useful for radiosity purposes. This can lead to large $\mathbf{Y}_k$ and $\mathbf{V}_k$ matrices.

2.3. Relation between the error in the factorization of **RF** and the radiosity error

Following Fernández [7], when $\mathbf{RF}$ is substituted by $\mathbf{RF}+\Delta\mathbf{RF}$, the relative error of the radiosity B (Eq. (1)) is upper bounded:

$$\frac{\|\tilde{B}-B\|}{\|B\|} \leq \frac{\|\Delta\mathbf{RF}\|}{1-\|\mathbf{RF}\|}$$

Taking into consideration that $\|\Delta\mathbf{RF}\| \leq \varepsilon$, where ε is the expected error of the factorization, and using the 2-norm, another bound can be formulated:

$$\frac{\|\tilde{B}-B\|_2}{\|B\|_2} \leq \frac{\varepsilon}{1-\sigma_{max}} \tag{5}$$

where $0 < \sigma_{max} \leq 1$ is the largest singular value of $\mathbf{RF}$. This bound is larger than ε. Experimental results show that in common situations the relative error of the radiosity results can be orders of magnitude smaller.

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