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Technical Section

Q2 Symmetry aware embedding for shape correspondence ☆

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ABSTRACT

In this paper, we present symmetry-aware embedding for shape correspondence, which is robust against symmetric (left–right) flips and rotational (front–back) flips. Unlike previous embedding approaches that embed surfaces into a high-dimensional space, our technique is based on a low dimensional (3D) embedding. Our method can solve left–right flips by finding a 3D rigid transformation between two embedding surfaces without reflections. Using the global reflectional symmetry plane to align two surfaces, we can further reduce the problem to that of finding the rotation that corrects the signs of the front–back and up–down directions (there are four possible solutions). We exploit this simple problem formulation and alleviate the front–back flips, by explicitly comparing the front and back of embedding surfaces based on the global and local extrinsic shape characteristics. Consequently, reasonably accurate point-to-point correspondences can be established simply by performing the nearest neighbor search in our embedding space. Experimental results based on a shape correspondence benchmark showed that our method produces stable matching results.

1. Introduction

In the last decade, the non-rigid shape correspondence field has advanced remarkably and many sophisticated techniques emerged [1–6]. Early approaches gradually deform a surface into another in the original 3D space [2,3]. The methods based on graph matching solves an assignment problem by minimizing intrinsic distortions such as discrepancies in geodesic distances. Recent techniques [1,6–8] embed surfaces into a high-dimensional space using spectral embedding techniques, such as Isomap [9] and Laplacian eigenmaps [10], to transform an intrinsic shape matching problem into an easier extrinsic one. Another class of techniques [5,11] use conformal maps to evaluate intrinsic distortions.

Although the embedding approach requires the optimization of a high-dimensional linear transformation only, it introduces a few challenges that are difficult to overcome. First, the embedding approaches encounter the problem called symmetry (left–right) flips because the reflectional symmetry cannot be distinguished by intrinsic shape characteristics. Second, finding a map (linear transformation) between two sets of high-dimensional eigenbasis is not an easy problem especially when the surfaces start to deviate from isometry. Under the isometric assumption, a map can be approximated as a rigid transformation which is close to a diagonal matrix. However, if the surfaces involves non-isomeric deformations, eigenbasis often exhibit ordering flips and multiplicities, which means that we can hardly predict the structure and patterns of a map. One can imagine this optimization problem becomes harder as the dimension of eigenbasis gets higher.

On the other hand, the approaches based on conformal maps [5] are robust against left–right flips because conformal maps are orientation preserving. The more challenging problem is the rotational (front–back) flip. As reported in Blended Intrinsic Maps (BIM) [5], it is difficult to judge front–back flips by evaluating conformal energy which is not sensitive to the changes in extrinsic shape characteristics (curvatures).

In this paper, we propose a novel shape correspondence technique based on a low-dimensional embedding—*symmetry-aware embedding*. With the use of a low dimensional embedding, we can make the solution space of the shape correspondence problem much smaller than that of high-dimensional embedding techniques. Low-dimensional embedding has been widely employed in the shape retrieval field. We extend one of the popular low dimensional embedding techniques, the least squares multi-dimensional scalings (LS-MDS) algorithm [12]. This embedding is orientation-preserving and can avoid left–right flips. We devise a deformation algorithm that can align and symmetrize this embedding with respect to the reflectional symmetry plane even in the presence of imperfect symmetries and missing parts.

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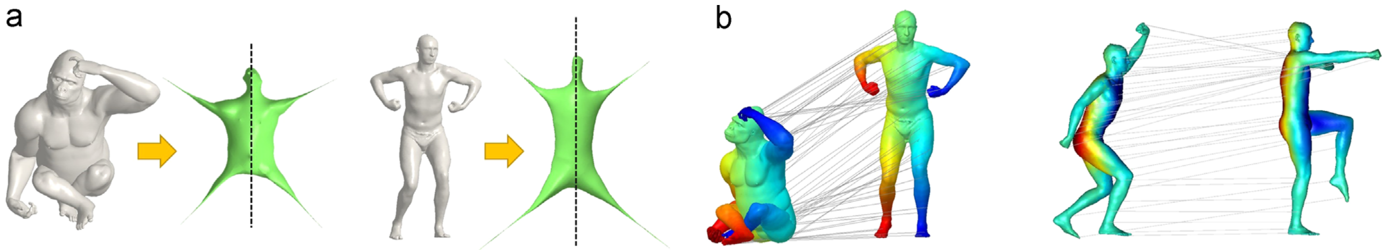

Fig. 1. Shape correspondence by symmetry aware embedding. Our method embeds surfaces in such a way that they are aligned according to the global reflectional symmetry plane (a). Reasonably accurate point-to-point correspondences can be established simply by performing the nearest neighbor search in the embedding space. Note that our method is robust against left–right and front–back flips (b).

Once we embed two surfaces with our symmetry-aware embedding, our task is to align them with each other up to local scales as in Fig. 1. Since our embedding is symmetrized, the correspondence problem is reduced to that of finding the best match from four possible solutions i.e., determining the signs of up–down and front–back directions. The front–back flip is thus explicitly tackled during this matching phase, by comparing the front and back of embedding surfaces based on the global and local extrinsic shape characteristics. After alignment, point-to-point correspondences can be established using the nearest neighbor search in the embedding space. Experimental results showed that our technique is more stable than the previous symmetry robust approaches [11,13], producing reasonably accurate correspondences.

The main contributions of this work are summarized as follows:

- We propose a low-dimensional (3D) embedding approach to shape correspondence. Although accuracy of shape correspondence is not as good as that of high-dimensional techniques, our low-dimensional approach can produce correspondences more stably. Our method can thus provide a good initial correspondences for high-dimensional embedding approaches.
- We provide a solution to alleviate the front–back flip problem. We compare the front and the back of embedding surfaces based on extrinsic shape characteristic, such as curvatures, to disambiguate the flip.
- We propose an intrinsic symmetry detection technique that is robust to imperfect symmetries and missing parts. It detects a global reflectional symmetry plane in 2D embedding space.

The rest of the paper is organized as follows: Section 2 briefly summarizes related work. Section 3 reviews and analyzes the previous embedding approaches, Isomap and LS-MDS. Section 4 describes the key idea and Section 5 shows the overview of our framework. The technique for constructing our symmetry-aware embedding is presented in Section 6. The shape correspondence algorithm based on our embedding is described in Section 7. We show experimental results in Section 8 and conclude in Section 9.

2. Related work

In the following, we will briefly review previous work on shape correspondence and symmetry detection. Please refer to well-organized surveys on shape correspondence [14] and symmetry detection [15].

Deformation-driven framework: Early approaches to non-rigid shape correspondence adopt the deformation-driven strategy. Li et al. [3] and Huang et al. [2] proposed local techniques that iteratively deform a template model using the as-rigid-as-possible deformation algorithm. In contrast, Zhang et al. [16] took a global approach that selects the best matching result from among all possible candidates of sparse correspondences, by non-rigidly deforming the source model and evaluating the distortions of the deformed results.

Intrinsic graph matching: Instead of deforming surfaces, many techniques seek for correspondences that minimize intrinsic distortions of two shapes. This can be formulated as a graph matching problem, which is a quadratic assignment problem (QAP). The main topic here is how to define the distortion measure such that it can be minimized efficiently while producing accurate correspondences. Bronstein et al. [17] used the Gromov–Hausdorff distance as their distortion measure in their generalized multi-dimensional scaling (GMDS) framework. Mobius voting by Lipman and Funkhouser [4] uses conformal maps. Kim et al. [5] proposed Blended Intrinsic Maps (BIM) that blends multiple conformal maps generated from sparse correspondences. Recently, a skeleton-based matching technique was proposed by Au et al. [18].

Embedding approach: Embedding approaches obtain two sets of basisfunctions for two shapes using such as Isomap [9] and Laplacian eigenmaps [10] and solve a linear assignment problem in the high-dimensional embedding space. Jain and Zhang [1] greedily matched eigenbasis from all possible combinations, ignoring multiplicity. Wuhrer et al. [19] used LS-MDS to improve the scalability of the spectral embedding method [1], starting from the solution of [1] as an initial. Mateus et al. [7] proposed eigenfunction signatures for matching eigenbasis, which is the histogram of the absolute values of eigenbasis. Ovsjanikov et al. [6] proposed functional maps that establishes correspondences of functions, instead of point-to-point correspondences, which improves robustness of matching in practice. Pokrass et al. [8] extended functional maps and enforced sparsity and diagonal constraints on a map as a strong isometric regularizer.

Symmetry detection: Symmetry has been studied extensively in computer vision and computer graphics for 2D images and 3D objects [15]. Podolak et al. [20] proposed a planar reflectional symmetry transform, a continuous measure of the reflectional symmetry. Mitra et al. [21] proposed a symmetry detection algorithm for discovering and extracting partial and approximate symmetries of 3D geometric models, based on local descriptors. They then proposed a symmetrization technique [22] for enhancing symmetries of the model based on this symmetry detection technique. Xu et al. [23] focused on partial intrinsic reflectional symmetries. Lipman et al. [24] analyze and represent symmetries in a point set based on their symmetry factored distance and embedding. Intrinsic symmetry detection has also been studied in the geometry field. Here, a self map that maps a surface to itself is detected. For example, previous approaches solve this problem based on GMDS [25], Laplacian eigenbasis [26] and Mobius Voting [27].

Symmetry-robust approaches: Recently, several correspondence techniques that are robust against symmetry flips are introduced. Liu et al. [11] used a symmetry axis to establish correspondences,

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