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Shape interior modeling and mass property optimization using ray-reps

Jun Wu [a,*], Lou Kramer [b], Rüdiger Westermann [b]

[a] Department of Mechanical Engineering, Technical University of Denmark, Lyngby, Denmark
[b] Department of Informatics, Technische Universität München, Garching, Germany

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ABSTRACT

We present a novel method for the modeling and optimization of the material distribution inside 3D shapes, such that their 3D printed replicas satisfy prescribed constraints regarding mass properties. In particular, we introduce an extension of ray-representation to shape interior modeling, and prove this parametrization covers the optimal interior regarding static and rotational stability criteria. This compact formulation thoroughly reduces the number of design variables compared to the general volumetric element-wise formulation. We demonstrate the effectiveness of our reduced formulation for optimizing shapes that stably float in liquids or spin around a prescribed axis.

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1. Introduction

Given the boundary surface representing the exterior of a 3D shape, we are interested in computing a material distribution in its interior, such that the resultant mass properties (i.e., the center of mass and the moment of inertia) satisfy a set of stability criteria. This design problem becomes very relevant in the era of 3D printing for fabricating customized shapes which stand [1,2], spin [3], or float [4,5] in a prescribed orientation.

A general formulation to this design problem is shape optimization on a volumetric basis [1,5]. To this end, the shape interior is discretized by hexahedral elements known as voxels, and each voxel is assigned a design variable. This parametrization results in a large number of design variables for accurately representing complex shapes.

To reduce the number of design variables, Bächer et al. [3] employ an adaptive octree grid where the cells split or merge on the fly during the optimization process. Musialski et al. [4] propose shape optimization by treating the thickness of the surface shell as design variables. Together with a reduced parametrization of offset surfaces, this formulation was demonstrated for static and rotational stability. However, the shell representation restricts the solution space, and thus an optimal solution that lies outside of this space cannot be reached.

In this paper, we present a reduced yet complete parametrization of shape interior, specifically designed for optimizing mass properties. Based on an analysis of mass properties and stability criteria, we prove that the optimal solution lies in a subspace represented compactly by an extension of ray-representations (ray-reps), which represent the shape by its intersections with parallel rays. In particular, we extend the concept of ray-reps by enhancing each ray with intervals distinguishing solid and void phases along this ray. The exact values of these intervals are automatically determined by an optimizer for controlling mass properties.

The specific contributions of our paper include:

- A compact parametrization of shape interior, which significantly reduces the number of design variables in shape interior optimization, and
- Insights into the optimal solution of mass properties under static and rotational stability criteria, which prove completeness of the reduced formulation, i.e., the optimal solution lies in the reduced space.

2. Ray-reps for shape interior modeling

We start by briefly introducing ray-reps for solid modeling, and then go on to the extended version for modeling the shape interior which can be partially void.

Ray-reps for solids: Ray-rep is a compact boundary representation, and it was introduced to the solid modeling community by Ellis et al. [6]. Similar approaches using the intersections between the surface mesh and parallel rays have been proposed, e.g., marching intersections [7], layered depth-normal images [8,9]. Ray-rep is

* Corresponding author. Tel.: +45 4525 4255; fax: +45 4525 1961.
E-mail address: junwu@mek.dtu.dk (J. Wu).

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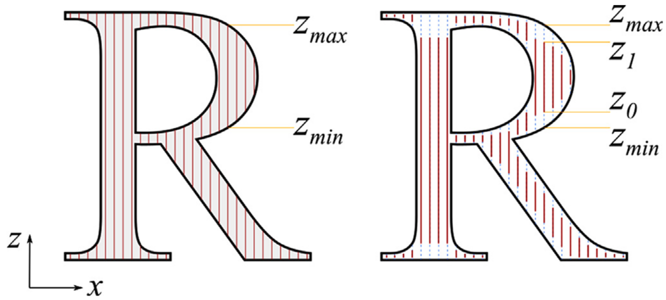


Fig. 1. Left: A *solid* of letter 'R' is represented by the intersections between its boundary (black) and the rays (red), i.e., $z_{\min}$ and $z_{\max}$ for each pair of intersections. Right: Two design variables z_0 and z_1 between each pair of intersections are introduced for modeling shape interior which is not necessarily fully solid. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

based on the parametrization of an object's surface using a set of parallel rays starting at a 2D grid. The solid is then represented by the sequence of intersections between its boundary surface and the rays. We assume rays start from a uniform grid on the *xy*-plane, and pierce along the *z*-axis which is aligned with the intended upright direction of the solid. As illustrated in Fig. 1 (left) on a 2D R-shaped solid, each ray has an even number of intersections with the watertight boundary surface. For each pair of intersections, we denote $z_{\min}$ to indicate the ray enters the solid, and $z_{\max}$ the ray exits the solid. We then collect all pairs of intersections by

$$\text{Ray}^i_{solid} = \{x^i, y^i, z^i_{\min}, z^i_{\max}\}, \quad (1)$$

where x^i and y^i are the *xy*-coordinates of the ray origin.

Rap-reps for shape interior: To extend ray-reps to shape interior modeling, we need intervals distinguishing solid and void regions along each ray. As will be proven in the next section, between each pair of intersections, there exists at most one solid region in the optimal solution under static and rotational stability criteria. Therefore we introduce two design variables z^i_0 and z^i_1, with $z^i_{\min} \le z^i_0 \le z^i_1 \le z^i_{\max}$, to represent the inner solid region. The ray-rep for shape interior is

$$\text{Ray}^i_{interior} = \{x^i, y^i, z^i_{\min}, z^i_{\max}, z^i_0, z^i_1\}. \quad (2)$$

Here $\overline{z^i_0 z^i_1}$ is solid while both $\overline{z^i_{\min} z^i_0}$ and $\overline{z^i_1 z^i_{\max}}$ are void. In the extreme case of $z^i_0 = z^i_1$, this ray is fully void; while in the extreme case of $z^i_{\min} = z^i_0$ and $z^i_1 = z^i_{\max}$, the ray is fully solid. The values of z^i_0 and z^i_1 are to be determined by the optimizer. We note that a similar extension of ray-reps for modeling heterogeneous materials has been presented by Wang [10].

This representation inherits the compactness and simplicity of original ray-reps, and thus provides a reduced parametrization for shape optimization as we will explore.

3. Optimizing mass properties on ray-reps

3.1. Mass properties

The stability of an object is determined by its mass properties, which include the total mass m, the center of mass $c = (c_x, c_y, c_z)^T$, and the 3×3 symmetric moment of inertia $I = (I_{xx}\ I_{xy}\ I_{xz}; I_{yx}\ I_{yy}\ I_{yz}; I_{zx}\ I_{zy}\ I_{zz})^T$. The mass properties are given by volume integrals over the domain of the object. With the ray-reps for shape interior, and assuming a constant material density ρ for the solid parts, the volume integrals are expressed as summation of line integrals over all rays:

$$m = \rho A \sum_i \int_{z^i_0}^{z^i_1} dz,$$

$$c_t = \frac{\rho A}{m} \sum_i \int_{z^i_0}^{z^i_1} t dz, \quad t \in \{x, y, z\},$$

$$I_{tt} = \rho A \sum_i \int_{z^i_0}^{z^i_1} (u^2 + v^2) dz, \quad \{t, u, v\} = \{x, y, z\},$$

$$I_{uv} = I_{vu} = -\rho A \sum_i \int_{z^i_0}^{z^i_1} uv dz, \quad \{u, v\} \subset \{x, y, z\},$$

where A is the area represented by a ray in the *xy*-plane. A is constant since the rays are sampled uniformly, and its value depends on the sampling resolution.

3.2. Static stability

The static status of standing and floating of an object is related to its center of mass. Here we explain our formulation and prove its completeness on floating, while an extension to standing can be easily derived accordingly.

The object floats if its buoyancy balances the gravity, and its center of mass should be at its lowest possible position, in order to maximize floating stability. For simplicity, we choose a coordinate frame such that the *z*-axis coincides with the intended upright direction. The design problem is formulated as

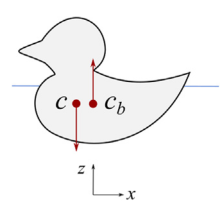
$$\underset{z_0, z_1}{\text{minimize}} \quad c_z, \quad (3)$$

$$\text{subject to} \quad (c_x, c_y) = (c_{b,x}, c_{b,y}), \quad (4)$$

$$m = m_b, \quad (5)$$

$$z^i_{\min} \le z^i_0 \le z^i_1 \le z^i_{\max}, \forall i, \quad (6)$$

where $(c_{b,x}, c_{b,y})$ is the *xy*-coordinates of the buoyant center, i.e., the centroid of the displaced volume of fluid, and m_b is the mass of the displaced fluid. We postpone the calculation of c_b, m_b, and the verification of floating stability to Section 3.4.



This reduced formulation is said to be *complete* in the sense that the optimal solution in the general volumetric element-wise formulation lies within the space defined by the above constraints on ray-reps.

Proof of completeness: Suppose in the volumetric element-wise formulation, the optimal solution contains two separated solid segments between a pair of intersections, denoted by $\overline{z^i_a z^i_b}$ and $\overline{z^i_c z^i_d}$, with the order $z^i_{\min} \le z^i_a < z^i_b < z^i_c < z^i_d \le z^i_{\max}$, see Fig. 2 (left). We can easily find a replacement of these two solid segments by one solid segment $\overline{z^i_0 z^i_1}$, with $z^i_0 = z^i_a$, and $z^i_1 = z^i_0 + z^i_b - z^i_a + z^i_d - z^i_c$, see Fig. 2 (right). This replacement does not alter the constraint Eqs. (4) and (5), i.e., c_x, c_y, nor m, but reduces the objective Eq. (3), i.e., c_z. This basically means that the optimal solution should contain at most one solid segment along each ray, since otherwise it can be further optimized by this replacement.

Actually, in static stability, this formulation can be further simplified by $z^i_0 = z^i_{\min}$, i.e., the solid segment, if exists, starts always from the lowest position. This is in line with the objective to minimize c_z. This fact is employed to further reduce the number of design variables by one half.

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