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B-spline surface fitting with knot position optimization

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ABSTRACT

In linear least squares fitting of B-spline surfaces, the choice of knot vector is essentially important to the quality of the approximating surface. In this paper, a heuristic criterion for optimal knot positions in the fitting problem is formulated as an optimization problem according to the geometric feature distribution of the input data. Then, the coordinate descent algorithm is used for the optimal knot computation. Based on knot position optimization, an iterative surface fitting framework is developed, which adaptively introduces more knot isolines passing through the regions with more complex geometry or large fitting errors. Hence, the approximation quality of the reconstructed surface is progressively improved up to a pre-specified threshold. We test several models to demonstrate the efficacy of our method in fitting surface with distinct geometric features. Different from the knot placement technique (NKTP method) proposed in Piegl and Tiller [1] and the dominant-column-based fitting method (DOM-based method) (Park [2]) which require input data in semi-grid or grid form, our algorithm takes more general data points as input, i.e., any scattered data sets with parameterization. Comparing to NKTP method and DOM-based method, our method efficiently produces more accurate results by using the same number of knots.

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1. Introduction

Fitting scattered data sets by surface with continuous representation is a frequently encountered problem in the fields of geometric modeling, geometric processing, reverse engineering and computer vision, and so on. The major mathematical tools used for continuous surface representation include implicit surface, subdivision surface, and parametric spline surface. Among them, parametric splines enjoy extensive popularity due to their excellent properties, such as explicit mathematical expression, convenience for derivative and integral and strong geometric intuition. In the last few decades, a large number of works were devoted to theoretical study or algorithm development for surface fitting problem with different splines, e.g., classical B-splines and Bézier basis [3], radial basis functions [4], T-splines [5,6], Delaunay Configuration B-splines [7,8], Triangular B-splines [9], spherical volumetric simplex splines [10] and manifold splines [11,12]. Among them, B-splines possess attractive properties, such as local control, convex hull and optimal continuity, and they have become one of the industry standards for shape modeling. Hence, B-spline surface fitting is still a central topic of computer aided geometric design and has been extensively studied.

In B-spline surface fitting, the data points are usually approximated by using a least squares formulation with respect to parameterization of input data, knot vectors and control points of B-splines. Hence, the parameterization, knot vectors, and control points all affect the fitting results. Much of the literature has focused on estimating these variables. In particular, knot vectors delicately influence the quality of approximation and the numerical stability of the system of the least squares problem. The choice of knot vector is crucial in effectively bounding approximation error to a given threshold. However, the existing methods on estimating the number of knots and their distribution are unsatisfactory, especially when the data set has unevenly distributed geometric features.

In this paper, we present an iterative algorithm that approximates scattered data points with a B-spline surface up to a pre-specified threshold. One of our contributions is the introduction of a heuristic criterion for optimal knot distribution in B-spline surface fitting. And, an efficient descent-based optimization algorithm is tailored for the knot position optimization. Based on the criterion, knots are adaptively added such that more knot isolines pass through the region with high geometric feature measures or

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fitting errors, hence more control points are introduced to the corresponding area on the surface. Unlike many existing methods that only apply to grid points, our method works for scattered data as long as its parameterization is available. The B-spline surface generated by our algorithm has a reasonable number of control points and is a highly accurate approximation to the input data. Compared with conventional knot selection strategies based fitting method, such as NKTP method [1] and DOM-based method [2], our method can efficiently generate approximating surface with higher accuracy.

2. Related work

The problem of approximating a given set of data points by a B-spline surface is a fundamental problem and has been widely investigated in fields of computer aided geometry design and computer graphics. Here, we will only point to some results which are closely related to our algorithm to give a background for our work.

The common approach approximates the data points in the least squares sense. In general, the following three issues must be dealt with: parameterization of the input scattered data, placement of knots, and choice of control points, all of them are essentially important to the quality of the approximating surface. Typically, if the parameters are given or pre-computed from the input data and the knot vectors are already determined, the control points become the unknowns of a quadratic function and can be obtained by solving a linear system of equations. However, it is usually a difficult task to estimate the parameters or knot vectors. Therefore, many research works focus on seeking appropriate parameterization or/and knot placement for the least square fitting problem.

In the earlier work, the input scattered data are assumed to be evenly distributed and organized in grid or semi-grid form. Uniform or chordal length parameterization is satisfactory for generating parameter points. Then, knots are chosen as the averages of certain number of consecutive parameters [3,13]. The approximating surface may be wiggly when the number of data points is close to the number of control points [1]. To overcome this issue, Piegl and Tiller presented an improved knot placement algorithm (NKTP), where knots are the averages of representative values of parameter point groups [1]. These previous method selected knots in a trivial manner such that each knot interval contains almost the same number of parameter values, which make them difficult to achieve adaptive fitting [2]. In our experiment, we find that the NKPT method places inadequate knots in the regions with sharply varied shape, hence the number of control point is not sufficient to reconstruct the geometric features.

To obtain more accurate approximation results, a straightforward method is to minimize the least squares problem by treating the parameters, knots and control point as unknowns. In that case, the least squares problem becomes highly nonlinear and one usually resorts to artificial intelligence methods for addressing this optimization task, such as genetic algorithm [14,15], artificial immune system algorithm [16], simulated annealing algorithm [17], estimation of distribution algorithms [18] and particle swarm optimization method [19]. Most of these methods focus on solving the fitting problem with B-spline curves, and their effective generalization to surface fitting is unavailable. Xie and Qin [20,21] developed interactive approaches for NURBS curve/surface fitting, where parameters, knot vectors and control points are optimized by conjugate gradient method. Besides simultaneous search of all the optimal parameters, iterative algorithms are also proposed to sequentially optimize subsets of the above quantities. For example, iterative algorithms are proposed to alternatively optimize the parameters and knot vectors for NURBS or B-spline surface fitting in [22,23]. A B-spline surface fitting algorithm for approximating point clouds with irregular topology was proposed in [24], where steps of reparameterization and knot refinement are iteratively applied. These global optimization algorithms could result in approximations with high fitting quality, but they are computationally expensive in finding global/local optima (in minutes or even hours). In addition, the number of knots usually needs to be determined in advance for most mentioned methods; but in practice, when a fitting tolerance is given, a simple guess before optimization on this number could easily end up to be either too many or too few.

A class of more efficient fitting strategies is to optimize parameterization only. For instance, Ma and Kruth [25] proposed an approach to assign parameter values to randomly measured points for B-spline surface fitting. Floater [26] presented a global and shape-preserving parameterization for surface triangulations for the purpose of well-behaved smooth surface interpolation and approximation. Lai et al. [27] proposed a feature sensitive parameterization approach, which assigns more parameter space to regions with higher geometric feature measures and thus benefits uniform B-spline surface fitting. Based on new fitting error terms, fitting techniques without parameterization are proposed in [28–30]. Alternatively, one assumes that parameterization is not subject to the optimization and focuses on knot placement. As the least squares fitting formulation is highly nonlinear with respect to the knot positions and the number of knots is unknown to obtain an approximant with desired quality, knots are usually placed heuristically and/or adaptively. For example, an adaptive knot placement algorithm for B-spline curve approximation to dense and noisy data points is proposed in [31]. Park and Lee [32] proposed an approach that selects dominant points from the input parameter points to generate knots for B-spline curve fitting. Later, this idea is generalized to choose dominant-column from the grid data points to obtain knot vectors for the B-spline surface fitting [2]. However, the dominant-column-based method (DOM-based method) assumes input data be rectangular grid points, thus, it is not applicable to data in more general forms. A similar problem for optimizing knot spacings for subdivision surface has also addressed in [33].

3. B-spline notations

Due to their favorable properties, B-splines have become well-known mathematical tools in applications in fields such computer aided design and computer graphics, and their theory has been extensively and exhaustively studied in much of the literature. We assume that the readers are already familiar with the basic concepts of B-spline as can be found, for example, in [34]. In this section, we only introduce notations of B-splines for the convenience of subsequent discussion.

Given knot vectors $U=\{u_{-m}=\cdots=u_0=0, u_1,\ldots,u_{p-1}, u_p=\cdots=u_{p+m}=1\}$ and $V=\{v_{-n}=\cdots=v_0=0, v_1\cdots, v_{q-1}, v_q=\cdots=v_{q+n}=1\}$ in u-direction and v-direction, respectively, a parametric tensor B-spline surface of degrees (m,n) is defined as follows:

$$S(u,v)=\sum_{i=0}^{p+m-1}\sum_{j=0}^{q+n-1} B_{i,m}(u)B_{j,n}(v)c_{ij}, (u,v)\in\Omega=[0,1]\times[0,1] \tag{1}$$

where $B_{i,m}(u)$ and $B_{j,n}(v)$ are the B-spline basis functions of degree m and n in u-direction and v-direction, respectively. c_{ij} for $0\le i\le p+m-1$ and $0\le j\le q+n-1$ are the control points and the mesh formed by them is called the control mesh. In this paper, we only concentrate on B-spline surfaces which are clamped in both parametric directions, i.e., the first and last m (resp. n) knots are

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