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Technical Section

Low budget and high fidelity relaxed 567-remeshing

Vincent Vidal [a,*], Guillaume Lavoué [b], Florent Dupont [a]

[a] Université de Lyon, CNRS, Université Lyon 1, LIRIS, UMR5205, Villeurbanne F-69622, France
[b] Université de Lyon, CNRS, INSA-Lyon, LIRIS, UMR5205, Villeurbanne F-69621, France

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ABSTRACT

567-meshes are a new type of closed and 2-manifold triangular meshes introduced by Aghdaii et al. in 2012 [1]. Vertices with valence less than 5 or greater than 7 are problematic for many mesh processing tasks such as edge collapse or surface subdivision. However, vertices with valence equal to 6 everywhere is most often impossible due to either surface topology or surface feature preservation, that is why 567-meshes are particularly of interest.

This paper proposes a 567-remeshing algorithm that locally retriangulates the mesh considering vertex valence, vertex budget and mesh fidelity as a whole. This algorithm also offers the possibility to preserve a set of feature edges during remeshing. This results in a framework capable of low budget 567-remeshing where remeshed models have a much higher fidelity to the original surface compared to the state of the art.

As applications of this work, we demonstrate that our remeshing improves the performances of mesh regularization and mesh connectivity compression.

1. Introduction

Non-regular triangular meshes are common place and their low and high valence vertices raise several issues. For instance, Aghdaii et al. [1] mention that valence-3 vertices may cause an edge collapse to generate a non-manifold mesh and that high valence vertices can lead to visible artifacts during mesh subdivision (e.g. for the butterfly scheme [2]). In addition, irregular valences, especially high ones, result in irregular sampling when applying either Laplacian or angle-based smoothing [3,4]. In particular, Surazhsky and Gotsman [3] noticed that edges whose two vertices have a valence greater than 7 (resp. smaller than 5) are made longer (resp. shorter) after an angle-based smoothing. The same authors [4] also mention that angle-based smoothing produces less inverted elements when the mesh is close to regular, which effectively happens for 567-*meshes*. 567-meshes are closed and 2-manifold triangular meshes, whose vertex valence is either 5, 6 or 7 [1]. In the remainder of this paper, k^- and l^+ denote a vertex with a valence, respectively, strictly less than k and strictly greater than l.

A regular mesh has faces only of the same degree and vertices only of the same valence. Completely regular (closed) meshes with only valence-6 vertices exist only for genus $g=1$ [5]. It is not possible to get a completely regular closed manifold of genus 0 [6]. In addition, either regular or semi-regular remeshing algorithms usually need a costly 2D (global) parametrization [7] which generally introduces some distortion.

Highly regular meshes are necessary for engineers performing numerical simulations [3]. Many authors [1,3,8,9] have developed (complex) strategies to either drastically reduce the number of non-regular vertices or at least avoid too irregular ones (5^- and 7^+). 567-*remeshing* algorithm [1] falls in the later category and has gain particular attention since it can resolve all issues mentioned in the first paragraph. In particular, a 567-remeshing step applied before a mesh smoothing greatly increases the final quality of the remeshed model. Note that 567-remeshed models could benefit from a specific data structure for representing adjacency relationships with constant-size buffers instead of linked-lists.

However, there is actually a costly counterpart with 567-remeshing: the decrease in the amplitude of valence irregularity is offset by an increase in the number of irregular and regular vertices. That is due by essence to *vertex split* operations that add vertices and to a theorem [1] that states that a valence $d>7$ vertex can be replaced by $\lfloor (d-2)/3 \rfloor - 1$ valence-7 vertices plus a valence 5 or 6 or 7 vertex, while incrementing the valence of $2(\lfloor (d-2)/3 \rfloor - 1)$ one-ring vertices by at most one. The current reference 567-remeshing algorithm [1] does a 1–9 triangle subdivision (see Fig. 1 (b)) before applying vertex splits to avoid creating a new valence-7^+ vertex. The resulting 567-remeshed models have their number of faces multiplied by about 10 before the algorithm' vertex removal step. And because the vertex removal step must preserve the 567 property and the fidelity to

* Corresponding author. Tel.: +33 4 72 43 13 38; fax: +33 4 72 43 15 36.
E-mail address: vincent.vidal@liris.cnrs.fr (V. Vidal).

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0097-8493/

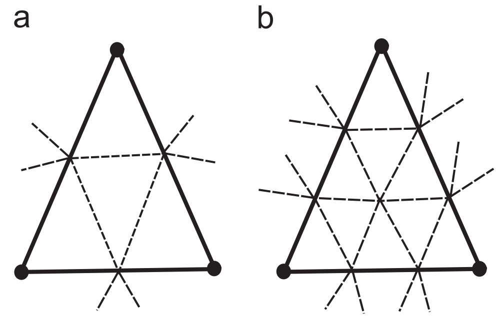

Fig. 1. Local triangle subdivision schemes whose the 1–9 scheme used in [1]: (a) 1–4 subdivision; (b) 1–9 subdivision.

the initial 567-remeshed model, it rarely counterbalances all added vertices, which thus becomes a severe issue when processing large meshes.

This paper addresses the vertex budget issue of the 567-remeshing introduced in [1] and proposes a framework also capable of controlling the geometric error and preserving mesh features (e.g. feature edges) during all the 567-remeshing operations. Our main contributions are the following

- *New local strategies for the removal of valence* 5^- *and* 7^+ *vertices*: We only consider local remeshing strategies (no global triangle subdivision) and we minimize the valence potential increase (see Eq. (1)) when we remove irregular vertices, by greedily selecting the local configuration associated with less valence potential increase.
- *Better control of the mesh quality*: Additionally to the vertex valences, the vertex budget and the fidelity to the original surface are monitored during the removal of valence 5^- or 7^+ vertices. An additional criterion, the triangle equilateralness, is also taken into account during the final mesh enhancement steps.
- *Preservation of lines of feature edges during all 567-remeshing operations*: Our framework offers the possibility to preserve a set of feature edges during 567-remeshing. It considerably improves the fidelity to the original surface, even for non-CAD models.

2. Related work

This section presents some works related to vertex valence optimization and the Aghdaii et al. [1] 567-remeshing algorithm published recently.

2.1. Minimal number of valence 5 and 7 vertices

When dealing with vertex regularization, it is a good start to know what is the theoretically minimal number of irregular vertices for 567-triangular models of closed 2-manifold meshes of arbitrary genus. Using the Euler characteristic for a 567-remeshed model we get $n_5 - n_7 = 12(1-g)$ [1] where g is the genus of the mesh and n_i denotes the number of valence-i vertices. It means that, when considering only topological aspects, the minimal number of valence-5 and valence-7 vertices for a *genus 0* closed 2-manifold mesh is 12 and 0 (the icosahedron), it is 0 and 0 for *genus* $g=1$ (the torus) and 0 and $12(g-1)$ for *genus* $g>1$ ($g=2$: we are able to get a eight mesh with 0 and 12; $g>2$: by cutting an handle along a circle that does not pass through any vertex and then reassembling like in [5]). These values of minimum number of valence-5 and valence-7 vertices may help for identifying the potential decrease of the number of irregular vertices within a 567-remeshed model. However, care should be taken to preserve the geometric fidelity to original surface, since each topological operation may potentially degrade it.

2.2. Connectivity regularization

Regularizing the connectivity of a given 2-manifold mesh M usually means minimizing the following functional [3]:

$$E(M) = \sum_{v \in M} \left(d(v) - d_{opt}(v)\right)^2 \quad (1)$$

where $d(v)$ is the valence of a vertex v and $d_{opt}(v)$ its optimal/ideal valence (6 for closed triangular 2-manifold meshes). This functional owns many local minima and is thus hard to globally minimize. Simulated annealing could be used for minimizing such a function, but its convergence towards a good local minima is too slow for practical purpose. In addition, the more local topological operations are applied on the mesh, the more its intrinsic geometrical features are lost. That is why an approach that uses the less as possible local operations to decrease the energy should be used. Surazhsky and Gotsman [3] have proposed an interesting strategy to further minimize the energy $E(M)$ starting from a local minima, by flipping towards each other edges made of one valence-6^- vertex and one valence-6^+ vertex up to they allow an additional energy decreasing flip. However, their algorithm still performs too many intermediate local modifications and thus degrades significantly the geometry. Moreover, since they forbid too degrading local operations, there is no guaranty that a good minima of the energy $E(M)$ is reached. A solution to better preserve the original geometry can be to severely limit the geometric error introduced during all local modifications such as in [8], but the resulting minimum of energy $E(M)$ is worst.

567-remeshing [1,10,11] has an advantage over general regularization algorithm such as the one presented in the last paragraph: the elimination of valence 3, 4 and 7^+ is a simpler problem than the elimination of all valence $\neq 6$ vertices. In particular, valence 3 and 4 vertices can be treated by a local retriangulation scheme (without geometric error), and valence-7^+ vertices can be tackled by well-chosen vertex splits. It means that a 567 regularization (local) is faster than the 6 regularization (global). In addition, the 567 regularization gives guarantee that no more vertex valence is out of 567, while the 6 regularization cannot provide this kind of guarantee. 567 regularization methods thus appear among the best connectivity regularization techniques, because they offer efficiency (running time and mesh quality) and guarantees.

In the following sections "*567 preserving*" means that it does not introduce neither any new valence-5^- nor any new valence-7^+ vertex.

2.3. Aghdaii et al. 567-remeshing algorithm

Aghdaii et al. [1] have introduced the concept of 567-meshes and proposed the following 567-remeshing algorithm:

1. *Firstly, all valence-*5^- *vertices are eliminated* without introducing geometrical error through the use of local retriangulation schemes (see Fig. 2). This step increases the valence of some vertices within the two-ring neighborhood of a former valence-5^- vertex. It can therefore add new valence-7^+ vertices, and that is why this step is done before the valence-7^+ vertex removal.
2. *Secondly*, *all valence* $d>7$ *vertices are eliminated* through a 1–9 triangular mesh subdivision (see Fig. 1) followed by $\lfloor (d-2)/3 \rfloor - 1$