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Technical Section

Position-based simulation of continuous materials ☆

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ABSTRACT

We introduce a novel fast and robust simulation method for deformable solids that supports complex physical effects like lateral contraction, anisotropy or elastoplasticity. Our method uses a continuum-based formulation to compute strain and bending energies for two- and three-dimensional bodies. In contrast to previous work, we do not determine forces to reduce these potential energies, instead we use a position-based approach. This combination of a continuum-based formulation with a position-based method enables us to keep the simulation algorithm stable, fast and controllable while providing the ability to simulate complex physical phenomena lacking in former position-based approaches. We demonstrate how to simulate cloth and volumetric bodies with lateral contraction, bending, plasticity as well as anisotropy and proof robustness even in case of degenerate or inverted elements. Due to the continuous material model of our method further physical phenomena like fracture or viscoelasticity can be easily implemented using already existing approaches. Furthermore, a combination with other geometrically motivated methods is possible.

1. Introduction

The physically based simulation of deformable solids is a topic of active research in the field of computer graphics for more than two decades. In general, simulation methods should reflect the behavior of real materials accurately to achieve realistic results. For this reason continuum mechanical approaches in combination with finite element methods (FEM) are widely used. These methods rely on either implicit time integration schemes or very small time steps to provide a robust simulation. Besides the accurate material behavior, many real-time applications focus on robustness and interactivity. Therefore, geometrically motivated approaches were developed to generate stable simulations while keeping computation times low. These methods result in a physically plausible behavior of the solid, but are not able to model complex material properties. Due to the robustness of the methods, they are often used to improve traditional approaches, e.g. strain-limiting methods for cloth simulations.

In this paper we present a novel fast and robust method for the simulation of two- and three-dimensional solids that supports complex physical phenomena. Our approach combines continuum mechanical material models with a position-based energy reduction. Its position-based nature allows us to perform a stable simulation using an explicit time integration scheme. Our method also strongly benefits from the continuous model since in contrast to previous position-based approaches it can handle complex physical effects like isotropic and anisotropic elastic behavior as well as the effects of lateral contraction. Moreover, we show how the problem of element inversion can be handled by only a small modification of the algorithm and provide a formulation to easily embed elastoplastic effects into the simulation. Finally, we demonstrate that our method is able to simulate thousands of degrees of freedom at interactive rates which makes it well-suited for applications like computer games, special effects in movies and virtual reality.

2. Related work

In the last two decades the simulation of deformable solids has been a topic of active research. Nealen et al. [1] presented a general survey of simulation methods. The first physically based approaches to simulate deformable objects used continuum mechanical formulations, where the governing equations were discretized and solved using numerical integration. Terzopolous et al. [2] used a finite difference scheme to discretize their model based on a non-linear strain measure. Later, O'Brien et al. presented a finite element method for spatial discretization to model brittle [3] and ductile [4] fracture, where explicit time integration schemes were used. However, explicit schemes suffer from stability issues regarding stiff differential equations which forces the usage of very small time steps resulting in high computational costs. James and Pai [5] discretized the equations using a boundary element method and solved the system in a quasi-static

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context avoiding the usage of a time integration scheme. Unfortunately, dynamic effects are completely neglected making the model unsuitable for scenes with spatially unconstrained, freely moving objects. Since explicit integration methods used in early works suffer from instabilities, implicit time integration schemes became popular. These schemes are computationally more expensive than explicit ones as large, generally non-linear systems of equations have to be solved. However, they allow us to use larger time step sizes. Regarding the finite element method with linear Lagrangian shape functions, the underlying strain measure has to be linearized to keep the systems linear. However, a linearized strain leads to artifacts when deformable bodies are exposed to rotational motion. Etzmuß et al. [6] introduced a corotational method, where the linearized strain is measured in a local, non-rotated reference frame, obtained by computing a polar decomposition of the deformation gradient. Due to derivatives of the rotational parts this still yields a non-linear system of equations, but nevertheless they found the simulation to be sufficient by neglecting non-linear terms in force derivatives and performing only a single step of Newton's method. To efficiently solve the linear system of equations multigrid solvers have been employed by Georgii et al. [7] as well as Dick et al. [8], where a hierarchy of discretizations is used to improve the rate of convergence for either short- and long-wavelength components. At the same time GPU-based solvers were investigated to simulate complex models in real-time [8,9]. Another approach to speed up simulation time was made by Hecht et al. [10]. They solved the linear equation system using a sparse Cholesky factorization which is incrementally updated during the simulation yielding a very efficient computation of the solution. Furthermore, the corotated approach was generalized to a discontinuous Galerkin finite element method by Kaufmann et al. [11] to overcome the restrictions of conforming bases. This easily allows the usage of convex as well as non-convex polyhedral elements with simple polynomial shape functions. As previously mentioned the corotational approach ignores higher order derivatives in the stiffness matrix which finally leads to instabilities with large deformations. Therefore, McAdams et al. [12] solved the instabilities by proposing a new integration rule for hexahedral elements. Further, Georgii et al. [7] presented a new method to extract the rigid body motion using an energy minimization resulting in a more stable corotational formulation. When considering the higher order terms, the instabilities are minimized but at the cost of solving non-linear systems [13].

Similar to existing continuum-mechanical approaches we use a spatial finite element discretization. A vast majority of simulation methods employ a corotational approach. In contrast to that our method easily allows us to use arbitrary non-linear material models that do not rely on a polar decomposition. Moreover, our approach is based on a position-based energy reduction step which allows a stable simulation with an explicit integration scheme.

In the last few years position-based methods became popular since they are fast, robust and controllable while no implicit time integration is required. A general survey of position-based methods is presented by Bender et al. [14]. The geometrically motivated, meshless concept of shape matching was introduced by Müller et al. [15] to simulate deformable objects. Traditional forces are avoided in favor of position displacements to solve a geometric constraint. The goal positions are determined by minimizing the distance between the reference shape and the deformed shape of a body. This minimization process requires the computation of translational vectors for both shapes and of a rotation matrix by a polar decomposition. To enhance the efficiency of the method Rivers et al. [16] proposed a fast summation method on regular lattices. Later, Diziol et al. [17] generalized the fast summation approach for irregular structures and introduced a method to enforce incompressibility. Bender et al. [18] introduced a multi-resolution approach to enhance the convergence. Further, Müller et al. [19] added an orientation to each particle to increase the robustness of shape matching. Another geometrically motivated technique is the position-based dynamics approach introduced by Müller et al. [20]. The authors demonstrated how to simulate cloth models by iteratively solving geometric constraints. Later, the position-based method was extended in order to simulate fluids [21], rigid bodies [22] and elastic rods [23]. Stam [24] also used constraints to simulate the deformation of a solid body, where the main difference to the approach of Müller et al. is that the solver of Stam is velocity-based.

The approach presented in this work integrates seamlessly into the position-based framework of Müller et al., but uses a material model derived from continuum mechanics to enforce physical phenomena that previous formulations are lacking and is even able to resolve degenerate or inverted shapes.

The simulation of cloth is closely related to the simulation of two-dimensional solids except of additional out-of-plane forces that prevent the object from excessive bending. For an overview we refer to the survey of Magnenat-Thalmann and Volino [25]. Early approaches were based on mass–spring systems, e.g. [26]. However, the behavior is difficult to control since the spring stiffnesses are generally unknown for specific materials. Later, also finite element methods solving partial differential equations derived from continuum mechanics were applied to model cloth [6,27]. Specific material behavior can then be enforced easily since the model provides established parameter sets used in classical mechanics. Müller et al. [20,28] presented a position-based method for robust interactive simulations. An additional bending constraint model for this position-based approach was introduced by Kelager et al. [29]. Since pieces of cloth only accommodate small amounts of stretching, strain limiting methods were developed. Thomaszewski et al. [30] limit the maximal strain of generally biphasic, anisotropic materials based on a continuum mechanical deformation measure. Later, Wang et al. [31] proposed an isotropic strain limiting where the underlying deformation gradient is directly modified by using a singular value decomposition.

In our approach we combine the advantages of both the continuum mechanical and the position-based approach to maintain complex physical phenomena while keeping the simulation easy to implement, fast, robust and controllable.

3. Position-based energy reduction

In our work we simulate the elasticity of the bodies by reducing potential energy functions $E(\mathbf{x})$ that correspond to their deformation. This energy reduction is performed using a position-based approach [20]. The concept of this approach is introduced in the following while the required energy functions are presented in Section 4.

3.1. Overview

We use particle meshes to represent deformable bodies in our simulation. Each particle has a mass m, a position $\mathbf{x}$ and a velocity $\mathbf{v}$. Mass lumping is used to concentrate the mass of the model at the vertices of the mesh which yields a diagonal mass matrix $\mathbf{M}$.

Position-based simulation methods typically work in three steps [14]. First, a time integration step is performed for a particle model in order to obtain new locations of the particles. These locations are used as predicted positions which are modified in the second step in order to fulfill given position constraints. The definition of bilateral constraints yields a system of equations which is generally non-linear. Solving this system exactly would result in a completely stiff body. Therefore, Müller et al. [20] propose to use an iterative solver and to perform only a few iterations which yields an elastic behavior and allows a high