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Intrinsic parameterization and registration of graph constrained surfaces

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ABSTRACT

In this paper, we present an intrinsic method to compute the bijective registration between genus zero surfaces with consistent feature graphs. First, the graph constrained surfaces are mapped to canonical domains by an intrinsic harmonic map, which extends the mean value coordinate to graph constrained surfaces in a rigorous and consistent way. The feature graph on the 3D surface is straightened to a planar straight graph, which forms a convex subdivision of the canonical domain. The parameterization exists, and is unique and intrinsic to the surface and its feature graph. Then the 3D surfaces with consistent feature graphs are registered by matching the straightened graphs and their associated convex regions in the canonical domain by constrained harmonic maps. The method is theoretically rigorous, and computationally efficient and robust. The application of surface morphing on various surfaces and images demonstrates the efficiency and practicality of the proposed methods.

1. Introduction

Surface parameterization and registration play important roles in many geometry processing applications such as texture mapping, surface modeling, morphing, matching, and so on. In practice, feature landmarks are widely used, and play an important role in the above applications. Anatomical landmarks are used in medical image analysis applications, for example, facial symmetry curves in adolescent idiopathic scoliosis and autism diagnosis, and brain sulci landmarks in Alzheimer' disease diagnosis and brain morphometry analysis. The anatomical landmarks on the surfaces usually form a 3-connected graph with nodes and curvy edges (see Fig. 1), where the nodes are the feature points (e.g., eye and mouth corners, nose tips), and the curvy edges are the landmark contours and curves connecting the nodes (e.g., eye and mouth contours). The primary goal of our paper is to compute the intrinsic parameterization and registration of graph constrained surfaces.

Most methods only focus on surface parameterization and registration with feature point constraints [1–3] and curve constraints [4,5]. For a surface with feature graph, it is worthy to deal with the graph as a whole rather than split the graph into separate points and curves as the graph has both global and local information, and serves as a skeleton structure of the surface. To our best knowledge, not much attention has been paid to tackle surface parameterization and registration of surfaces with graph constraints. Recently, Zeng et al. [6] presented a method to parameterize surfaces with graph constraints, and register two 3D surfaces with consistent graphs. The parameterization method first compute the planar embedding of the feature graphs using the Tutte embedding, and then tries to compute a harmonic mapping to fit the graph embedding in the canonical domain. As a result, the final parameterization and registration highly depend on the chosen weights during the Tutte embedding computation, which is heuristic and not intrinsic.

In this paper, we present intrinsic parameterization and registration methods for graph constrained surfaces by extending the mean value coordinate adaptively according to the associated feature graphs. The parameterization presented in this paper provides an intrinsic representation for surfaces with feature graphs, where the curvy feature graph is straightened to a planar straight line graph (PSLG) in the canonical domain, and its shape is determined intrinsically by the surface geometry and its feature graphs. The parameterization is bijective, and has the guarantee of existence and uniqueness, based on which the canonical domains are registered bijectively by aligning the two consistent PSLGs using the constrained harmonic map. For two surfaces with same topological graphs, these graphs serve as guidance for the correspondence computation between them. The nodes of the source graph are mapped to the corresponding ones of the target graph while the interior points of the source graph can slide on the corresponding target graph curves, and their positions are computed automatically by the intrinsic registration method.

1.1. Approach overview

In discrete setting, 3D surface is represented as a triangular mesh denoted as $M = (V, E, F)$, where *V, E, F* represent vertex, edge, and face

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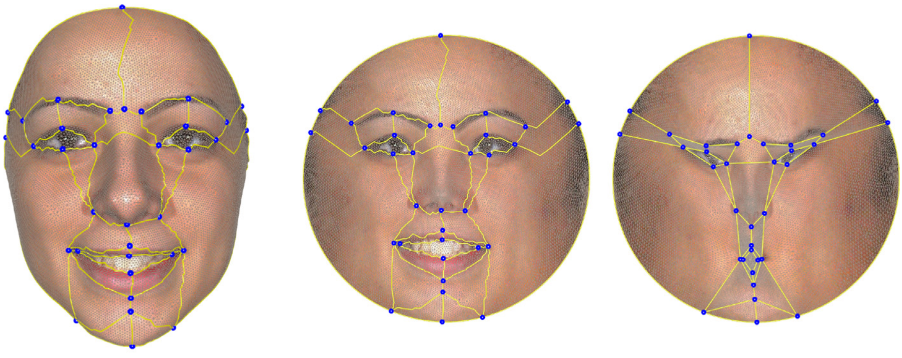

(a) 3D face (b) conformal map (c) intrinsic map

Fig. 1. Parameterization of 3D facial surfaces with feature graphs. Given a happy facial surface decorated with a 3-connected feature graph in (a), (b) and (c) shows the conformal parameterization and the intrinsic parameterization, respectively.

sets, respectively. The graphs can be extracted from natural surfaces automatically or manually [7], which are defined as 3-connected (i.e., each vertex connectivity ≥ 3) graphs. The nodes of these graphs are the dominant feature points of the surface, and the edges of the graphs are the curves between them. To distinguish with the vertex and edge of the mesh, we use *graph-node* and *graph-edge* to denote the node and edge of the graph in the remainder of the paper, respectively.

In this paper, we propose a method to compute the intrinsic harmonic map of graph constrained surfaces, which maps the feature graph on the surface to a PSLG in the parameter domain, and maintains the geometry of the original surface as much as possible (see Fig. 1). The harmonic weights are computed by applying the Circumferential Mean Value Theorem adaptively according to the associated feature graph. For vertices lying on the graph-edges, we adopt the *one-ring graph neighborhood* instead of the traditional *one-ring neighborhood* to derive the harmonic weights, which extends the mean value coordinate to graph constrained surfaces. The intrinsic parameterization is bijective and respects the feature graph constraints. To register two surfaces with consistent feature landmarks, the PSLGs and their associated convex subregions in the canonical domain are exactly aligned by a constrained harmonic map, which is unique and bijective. The registration method presented in this paper can be applied to generate morphing sequences between surfaces and images with consistent feature graphs, which demonstrate the efficiency and practicality of the proposed methods.

1.2. Related work

Surface parameterization. Surface parameterization was first introduced to computer graphics as a method for mapping texture onto surface [8,9] and has gradually become a useful tool for many geometry processing applications, such as detail mapping, synthesis and transfer, mesh editing and compression, remeshing, fitting, morphing, and so on [10,11]. Surface conformal mapping as the most popular surface parameterization method has its nice property, angle preserving, and has been widely used for various shape analysis applications [12]. It has been intensively studied over the last two decades, including the harmonic energy minimization [13], least square conformal maps [14], holomorphic differentials [15], discrete curvature flows [16–18], and so on [19–26]. As a general mapping, quasiconformal mapping has been arousing more and more attention recently [4,27]. The auxiliary metric method was presented with the 1-form and curvature flow methods [28,29]. The holomorphic Beltrami flow method [30] was introduced using a variational principle. In this paper, the quasiconformal mapping of graph constrained surfaces is computed intrinsically by an adaptive harmonic map, which can be formulated by sparse linear systems. The method is general, easy to implement and the entire process is automatic, which straightens the graph curves, and preserves local shape of the original surface.

Surface registration. In the past decade, 3D surface registration method have been intensively explored [31–41], which has a broad range of applications including shape matching and recognition, shape modeling, morphological study and animations. Most existing methods directly deal with non-rigid deformations, but always stop at a local optima and hardly get a global solution. A common approach is to find a parameterization of the two surfaces to a common base domain [40,41] and then define the final map by composing one parameterization with the inverse of the other. Schreiner et al. [41] used progressive meshes to define the base domain of the source and target surfaces and optimize the map between the surfaces, which is slow to converge and could easily get stuck in a local minimum. Praun et al. [40] constructs the base domain to compute the consistent mesh parameterization for a group of models. Panozzo et al. [39] used generalized weighted averages on surfaces to define mappings between surfaces. However the set of anchors must be carefully designed to cover the surface well, and the bijectivity is not guaranteed. Recently, a lot of research focuses on surface conformal and quasiconformal mapping based methods [42–46]. According to surface uniformization theorem [47], any arbitrary surface can be conformally mapped to one of three canonical domains, the unit sphere, the Euclidean plane or the hyperbolic disk. By mapping surfaces to 2D canonical domains, the problems of 3D surface registration is reduced to a 2D image registration problem. In real applications, landmark constraints are usually prescribed to guide the surface registration, which may introduce fold singularities in the resultant mapping. Among the various feature landmarks (points, curves, and graphs), the feature graph plays an important role in the constrained surface registration. For the surfaces with feature graph, it will introduce more benefits to treat the feature graph as a whole rather than split it into separate curves, and apply the traditional curve based methods [5,46]. It should be emphasized that all the above methods do not aim to produce bijective maps between graph constrained surfaces. Zeng et al. [6] presented the first method to parameterize and register surfaces with graph constraints. However, it determined the weights of the tutte embedding by a heuristic method. In this paper, the feature graph is intrinsically mapped to canonical shapes in the canonical domain, and the harmonic weights are determined by approximating the harmonic function using the Circumferential Mean Value Theorem adaptively according to the associated feature graph.

1.3. Contributions

The major contribution of this work is to present intrinsic methods to compute the parameterization and registration of graph constrained surfaces, which extends the geometric mapping based registration framework to deal with graph decorated surfaces intrinsically. In detail,

1) It presents an intrinsic parameterization of graph constrained surfaces, which maps the feature graph to a PSLG in the canonical domain. The method is unique and free of fold singularities.
2) It presents a bijective registration framework for graph constrained surfaces, which aligns the 3-connected graphs intrinsically. It can handle both simply-connected and multiply-connected domains.