Graphical Models 96 (2018) 38–46

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

# A retroreflective BRDF model based on prismatic sheeting and microfacet theory

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ARTICLE INFO

*Keywords:*
Retroreflection
Rough surface
Prismatic sheeting
Microfacet
BRDF

ABSTRACT

We propose a novel reflectance model for the physically based rendering of retroreflective materials, based on prismatic sheeting and microfacet theory. Due to its high performance, prismatic sheeting has been extensively used in material industry. We show through a geometric optics analysis that a prismatic sheet with a smooth incident plane exhibits perfect retroreflection, and retroreflectivity varies with respect to the incident direction. To permit imperfect retroreflection that appears frequently in many common situations, we model the incident plane as a rough surface with a microfacet profile. By adjusting surface roughness, we can easily change the angularity of the retroreflection lobe. We analyze thoroughly the relationship between surface roughness and glossiness of retroreflection using joint spherical warping strategy. Based on the analysis, a practical BRDF model is derived for materials with a prismatic sheet which comprises a surface reflection term, a retroreflection term, and a diffuse reflection term.

## 1. Introduction

Retroreflection is a special, yet common optical phenomenon in which reflected light preferentially returns in a cone back to the direction of the light source. Materials with such property are extensively used for safety garments to help enhance the visibility of the wearer in low-light conditions, and for traffic facilities to ensure highway safety of drivers during their nighttime travels. They are also essential for head-mounted projection displays in augmented environments [1,2], and for some 3D displays [3].

Although retroreflective materials are important and used widely in many situations, their reflectance behaviors are less-studied in Computer Graphics, probably due to the difficulties in acquiring measured reflectance data from real retroreflective materials using traditional gonioreflectometers [4,5]. Most existing BRDF models in Computer Graphics only capture surface reflectance that is dominated by forward scattering. In fact, accurately reproducing the appearance of retroreflective materials is challenging due to the complex geometric structures and the special lighting patterns found at the microscopic level.

A recent study by Belcour et al. [6] reveals that some existing parametric BRDF models can be easily updated to support retroreflection. They designed a special acquisition system to measure several isotropic retroreflective materials, and introduced a new parameterization with a back vector to effectively fit the data. Nevertheless, the proposed BRDF models are empirical and approximated with the parameters deviated from actual physical characteristics.

In this paper, we propose a physically motivated BRDF model dedicated to dealing with retroreflection. The key insight is to simulate each retroreflective surface with a dielectric prismatic (or cube corner) sheet [7,8], a widely adopted 3D geometric structure in material industry to achieve retroreflective effects. A detailed geometric optics analysis reveals that pronounced retroreflection is found when a light ray is successively reflected by three mutually perpendicular planes of the sheet. The portion of incident light that is successfully retroreflected is fully described by the effective retroreflective area — a function of incident direction.

In order to support imperfect retroreflection with a finite divergence [9,10], traditional prismatic sheeting is extended to contain a rough incident plane modeled with microfacet theory [11], in which the roughness is represented by a normal distribution function (NDF). We study theoretically and experimentally the effect of surface roughness on the directional distribution of retroreflected light, using the joint spherical warping strategy [12,13]. The study leads to an analytical formula for the retroreflection lobe, if the NDF follows a Beckmann distribution.

Based on the insight gained from the analysis, a novel BRDF model that simulates reflectance properties of prismatic sheeting is derived without the need to model surface details explicitly. This BRDF model comprises three types of reflection: a surface reflection term from the

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https://doi.org/10.1016/j.gmod.2018.01.002
Received 28 July 2017; Received in revised form 12 December 2017; Accepted 20 January 2018
Available online 08 February 2018
1524-0703/ 

rough incident plane of the sheet, a retroreflection term caused by large quantities of tiny prisms inside the sheet, and a diffuse reflection term from the substrate upon which the sheet is welded. We demonstrate that the proposed BRDF model is readily implemented into a physically-based rendering engine to generate correct retroreflection effects. Experimental results reveal that the proposed BRDF model is both physically plausible and computationally efficient with a convenient user control over several physical parameters.

## 2. Related work

In the following, we discuss the most relevant previous work on retroreflection and microfacet-based BSDF models.

### 2.1. Retroreflection

Very few works in Computer Graphics consider retroreflective materials, but several models exist in physics literature. Similar to ours, Trowbridge [14] theoretically studied the retroreflective property of light by cube corner structures and derived a BRDF model accounting for retroreflection from rough surfaces. Since this model is very sophisticated and lacks the Fresnel term, it is not suited for realistic rendering. Other models rely on light interaction with a glass beaded sheet [15–17] which is another technique capable of generating retroreflection by injecting glass beads into some strips. However, due to their numerical nature, these models are also hardly used in the rendering realm.

Besides the models of Belcour [6], there are also some empirical models that can be easily adapted to include retroreflection. The generalized cosine lobe model described in Lafortune [18] allows to define a retroreflective lobe by specifying proper coefficients of the lobe direction. Neumann [19] suggested some modifications and correction factors for several empirical models to make them physically plausible. The proposed new models can approximate many practical materials, including retroreflective paints.

Certain rough surfaces may produce grazing retroreflection to flatten the diffuse appearance, and such behavior can be predicted by the Oren–Nayar model [20]. Velvety or dusty surfaces exhibit an increase in reflection toward grazing angles due to a turbid layer covered on the surfaces [21]. These models are quite different to ours since they only capture special retroreflection at grazing angles.

### 2.2. Microfacet models

Our work relies on microfacet theory to represent surface roughness of a retroreflector. Microfacet theory was first introduced to Computer Graphics by Cook and Torrance [11] to quantify surface reflection of rough materials. It defines the BRDF as an aggregated response of an infinite number of tiny facets that act as ideal mirrors when radiated by light rays. Experimental validations against real-world measurements have consistently proven that microfacet-based BRDF models compare favorably with other families of parametric BRDF models [4,22–26]. In microfacet models, the microfacet orientations are statistically described by the NDF, which is crucial in the definition of a microfacet-based BRDF.

Besides surface reflection, microfacet theory has also been adopted to simulate transmitting effects. Stam [27] derived a microfacet-based BTDF model for refractive materials with rough surfaces. This model was further refined by Walter [28] with proper normalization and importance sampling strategies. Recently, microfacet theory has been extended to support subsurface reflection [12,13], multiple scattering [29], glints [30], and iridescence [31,32]. Our work also can be viewed as an extension to microfacet theory.

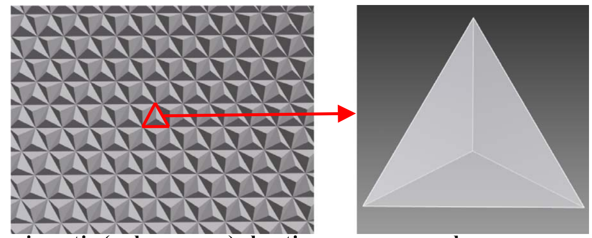


**Fig. 1.** The geometric structure of a prismatic sheet.

## 3. Retroreflector geometry

In this paper, we model retroreflective surfaces with prismatic sheeting. As schematically described in Fig. 1, a prismatic sheet is a close-packed array of cube corners, and each cube corner consists of three mutually perpendicular reflecting planes and an incident plane through which rays refract into the structure. Traditional prismatic sheeting assumes the incident plane to be smooth such that any retroreflected light is bent through 180° and exits the structure exactly along the inverse incident direction, resulting in perfect retroreflection. However, in many practical situations, especially in traffic signage and safety applications, we would like the outgoing light to spread in a wide angularity around the perfect retroreflection direction to allow some imperfect retroreflection [9,10]. This is quite helpful since in many cases the incident and viewing directions are not necessarily parallel. To make such imperfect retroreflection controllable, we introduce some imperfections to the ideal case by depicting the incident plane with the microfacet model in which the roughness is encoded in the NDF.

In this section, we first prove, based on the geometric optics analysis, that a cube corner with smooth incident plane can generate perfect retroreflection. Then, we expose the fact that not all incident light is properly retroreflected and an effective retroreflective area exists for a given cube corner which varies significantly with respect to the incident light direction. After that, we show in great details how the surface roughness of the incident plane alters the visually perceived glossiness of the imperfect retroreflection. We demonstrate that if the NDF of a rough surface follows a Beckmann distribution, we are able to find another Beckmann distribution which fits the retroreflection lobe quite well.

Currently, we assume that the size of the surface structure is much large than the wavelength of light. Therefore, wave effects such as diffraction and interference are not considered.

### 3.1. Smooth incident plane

In the cube corner with a smooth incident plane as shown in Fig. 2(a), an incident light ray with direction $\boldsymbol{\omega}_i$ may be refracted twice (e.g., at points **I** and **R**) and total internal reflected three times (e.g., at points **D, E**, and **F**) before it finally exits the structure along a ray with direction $\boldsymbol{\omega}_o$. If the incident plane $\triangle\mathbf{ABC}$ is smooth, we can prove that $\omega_o = -\omega_i$.

As evidenced in Fig. 2(a), the incident ray after first refraction (i.e., the orange ray $\overrightarrow{\mathbf{ID}}$) may be successively total internal reflected by three dielectric triangles $\triangle\mathbf{AOB}$, $\triangle\mathbf{BOC}$, and $\triangle\mathbf{COA}$, at three intersection points **D, E**, and **F**. The outgoing ray $\overrightarrow{\mathbf{FR}}$ before refraction is parallel to $\overrightarrow{\mathbf{ID}}$ with an opposite direction since the dihedral angle between each pair of adjacent reflecting triangles is exactly 90°. This makes $\omega_o = -\omega_i$ according to the principle of light reversibility.

Notably, to complete the retroreflection, the straight path along $\overrightarrow{\mathbf{ID}}$ should intersect triangles $\triangle\mathbf{BOC'}$ and $\triangle\mathbf{A'OC'}$ successfully at two inner cross points **E′** (mirror symmetric point of **E** about the $xy$ plane) and **F′** (mirror symmetric point of **F** about the origin). Otherwise, the incident ray will refract at some plane and exit the structure along a