Ad Hoc Networks 12 (2014) 243–249

Contents lists available at SciVerse ScienceDirect

Ad Hoc Networks

journal homepage: www.elsevier.com/locate/adhoc

Finding available parking spaces made easy ☆

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ARTICLE INFO

Article history:
Received 7 March 2011
Received in revised form 1 January 2012
Accepted 8 March 2012
Available online 17 March 2012

Keywords:
Continuous-time Markov chains
Matrix exponentials
Parking availability
Vehicular ad hoc networks

ABSTRACT

We discuss the problem of predicting the number of available parking spaces in a parking lot. The parking lot is modeled by a continuous-time Markov chain, following Caliskan, Barthels, Scheuermann, and Mauve. The parking lot regularly communicates the number of occupied spaces, capacity, arrival and parking rate through a vehicular network. The navigation system in the vehicle has to compute from these data the probability of an available parking space upon arrival. We derive a structural result that considerably simplifies the computation of the transition probabilities in the navigation system of the vehicle.

1. Introduction

A navigation system in a car allows the driver to find the destination with ease. Nowadays, most car manufacturers offer the option to install a navigation system, and aftermarket solutions are available as well. In addition to giving driving directions, navigation systems typically also provide a surprisingly accurate estimate of the remaining travel time. Upon arrival at the destination, the search for an available parking space begins.

In many urban or metropolitan areas it can be quite time consuming to find an available parking space. However, the parking problem is more than a nuisance. A recent article in the Washington Post [8] points out that:

> "Hunting for parking produces more than frustration. Shoup studied a 15-block business district in Los Angeles and determined that cruising about 2.5 times around the block for the average of 3.3 min required to find a space added up to 950,000 excess miles traveled, 47,000 gallons of gas wasted and 730 tons of carbon dioxide produced in the course of a year."

The recent advances in sensing and communication technology prompt the question whether vehicular networks can help reduce the time in the search for available parking spaces. There already exist various systems that give information about currently available parking spaces, see e.g. [1,7,10,11,15,16]. Furthermore, there exist pilot systems for metered street side parking [8,13], which monitor the occupancy of each parking space by a sensor. The high costs for sensor deployment and maintenance are recovered by aggressive automated ticketing of parking violations.

The monitoring of each single parking space is not economically sensible in parking lots of airports or malls. However, it is quite feasible to monitor the flow of entering and exiting vehicles to a parking lot of an airport or a mall. We envision a deployment scenario in which comparatively fewer sensors will provide information about the capacity, current occupancy, arrival rates, and parking rates. The information can be distributed through a vehicular ad hoc network or a mobile cellular network.

☆ This is an extended version of a paper that was published in the proceedings of DIALM-POMC'10.

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[1] The research by A.K. was supported in part by NSF Grants CCF 0622201 and CCF 1018500.

[2] The research by H.L. was supported in part by NSF Grant CNS-0614929.

[3] The research by J.W. was supported in part by NSF Grant 0500265 and Texas Higher Education Coordinating Board Grant ARP-00512-0007-2006.

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http://dx.doi.org/10.1016/j.adhoc.2012.03.002

A driver might have the following questions when approaching a parking lot:

(a) Will a parking space at a particular parking area be available when I get there in t minutes?
(b) If there is no parking space available, how long do I need to wait until a parking space becomes available?

One important criterion to consider in finding answers to the above questions is the timing factor when the spot is available. Obviously the data gathered using the vehicular network are always from the past. Thus, a natural question is: How can one *predict* the availability of some parking space in a reasonably correct and efficient way? We propose an online scheme that answers this question with a very small communication overhead.

2. Related work

There have been numerous works studying the problem of finding available parking spaces using a vehicular ad hoc network; those closely related to our work are [2,3,10–12].

Mathur et al. [11] motivate the problem of finding available parking spaces, discuss research challenges and propose some possible solutions. They discuss a centralized architecture and a distributed architecture for their solutions. In the centralized solution, some cars are equipped with ultrasonic sensors, which drive past the parking spaces to collect the occupancy data, and upload the data to the centralized database. The cars that need to park simply query the centralized database. The idea of this centralized architecture is deployed and experimented with in ParkNet [10]. The distributed architecture uses car-to-car communication and Mathur et al. [11] address the importance of having such a distributed architecture.

A network model that consists of onboard units on vehicles and static road side units is proposed by Panayappan and Trivedi [12] with an emphasis on secure communication between these units. In the paper by Caliskan, Graupner and Mauve [3], the parking area is modeled as a grid, and schemes for information aggregation and dissemination over the grid are proposed.

Our work studies how to use the information collected by such vehicular ad hoc networking systems, as the ones proposed above. Caliskan et al. [2] introduced a simple and elegant parking lot model that we adopt here. We review this model and give some background in Section 3. We then show how to explicitly calculate the transition probabilities, which was claimed to be a difficult problem by Caliskan et al. [2]. Our main result provides a factorization of the transition probability matrix that can be easily evaluated. The chief benefit of our solution is that it is suitable for implementation in the navigation system of vehicles.

3. The parking lot model

In this section, we review the parking lot model that was introduced by Caliskan et al. [2] and give the necessary background on continuous-time Markov chains.

Suppose that we are interested in parking at a parking lot with n parking spaces. Since we are only concerned with predicting whether or not the parking lot is full, it suffices to model the number of occupied parking spaces. For all times $t \geqslant 0$, let $X(t)$ denote a random variable with values in $\{0, 1, \dots, n\}$. We denote by $\Pr[X(t) = u]$ the probability that at time t there are precisely u parking spaces occupied. The future availability of the parking spaces depends on the present occupancy, but not really on past occupancies. Therefore, one can model the changes of the parking spots by a continuous-time Markov chain. By definition, the stochastic process $\{X(t)|t \geqslant 0\}$ is a continuous-time Markov chain if and only if

$$\begin{aligned}\Pr[X(t+s) = u|X(s) = v, X(s_1) = w_1, \dots, X(s_n) = w_n] \\ = \Pr[X(t+s) = u|X(s) = v]\end{aligned} \quad (1)$$

holds for all $n \geqslant 0$ and for all occupancy numbers u, v, $w_1, \dots, w_n$ in $\{0, 1, \dots, n\}$ and all nonnegative real numbers $s_1 < s_2 < \dots < s_n < s$ and t. This formalizes the notion that the future occupancy of the parking spots depends on the present occupancy but not on past occupancies. We will make a small excursion and introduce some terminology from the theory of continuous-time Markov chains.

The Markov chain is called homogeneous if the right-hand side of Eq. (1) does not depend on s. For simplicity, we will assume that our Markov chains are homogeneous. Furthermore, we will assume that the transition probabilities $\Pr[X(t+s) = u|X(s) = v]$ are right-continuous at $t = 0$, meaning that

$$\lim_{t\to 0+} \Pr[X(t+s) = u|X(s) = v] = \begin{cases} 1 & \text{if } u = v, \\ 0 & \text{if } u \neq v. \end{cases}$$

This assumption ensures that the Markov chain has with probability 1 only a finite number of state changes in small time intervals.

Let us define the probability transition matrices

$$P(t) = (\Pr[X(t+s) = u-1|X(s) = v-1])_{1\leqslant u,v\leqslant n+1}$$

for all real numbers $t \geqslant 0$. Notice that the indices of matrices and vectors consistently range from 1 to $n + 1$ in this paper.

Obviously, the behavior of the Markov chain is completely determined by the initial probability distribution on its states and the probability transition matrices. Some basic properties of the probability transition matrices are:

(a) $P(t)$ is a stochastic matrix for all $t \geqslant 0$, that is, each row of a probability transition matrix sums to 1.
(b) $P(r + s) = P(r)P(s)$ holds for all real numbers $r, s \geqslant 0$.
(c) $\lim_{h\to 0+} P(h) = P(0)$ is the identity matrix.

Therefore, $\{P(t)|t \geqslant 0\}$ is a continuous semigroup (recall that a semigroup is a set that is equipped with a binary associative operation). The continuity of this semigroup has the somewhat unexpected consequence that $P(t)$ is infinitely differentiable with respect to $t > 0$, see [9, p. 164]. Therefore,

$$Q = \lim_{h\to 0+} \frac{1}{h}(P(h) - I)$$