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Short communication

A closed-form sum-rate lower bound for amplify-and-forward multi-way relay networks

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ABSTRACT

In this letter, we investigate the sum-rate performance of the multi-way relay network (MWRN) with amplify-and-forward (AF) relaying, where multiple users exchange information with the help of a relay node. We consider a full exchange scenario where each user has to decode messages form all the other users. We analyze the sum-rate lower and upper bounds of the AF MWRN, and derive a closed-form expression for the average sum-rate low bound when the number of users is sufficient large. Numerical results show that the derived bound is tight.

1. Introduction

The two-way relay network [1–4] has been generalized to multi-way relay networks (MWRNs) recently [5,6], where multiple users exchange their information through a relay node. MWRN models a variety of communication scenarios like teleconferencing, message exchange in a social network or satellite communication. For instance, in cellular networks, a set of mobile users can form a social network and exchange information by communicating via the base station, which serves as the relay in the MWRN.

Compared with conventional multiuser cellular networks [7,8], the signal transmission and detection are more complicated in MWRNs. So far, various protocols have been proposed for MWRNs to achieve significantly improved spectral efficiency in wireless communication systems, e.g., decode-and-forward (DF) [5], compress-and-forward (CF) [5], amplify-and-forward(AF) [6], and complex field network coding [9].

In general, there are two data exchange models for the MWRN, namely, pairwise data exchange and full data exchange. For MWRNs with pairwise data exchange, the authors in [10] analyzed the degrees of freedom (DoF) of symmetric multiple-input multiple-output (MIMO) MWRNs with clustered pairwise data exchange. In [12], the authors investigated the optimal users' transmission orderings to achieve the maximum common rate and sum rate of pairwise MWRNs with functional-decode-forward relaying. For MWRNs with full data exchange, the authors in [6] investigated the achievable rate region for a DF MWRN, while [11] proposed lattice coding based DF relaying scheme for the MIMO MWRNs and derived the achievable rates.

Compared with DF relaying, AF based relaying has the advantages of simpler implementation and lower complexity at the relay. However, the performance analysis of such AF MWRNs is more challenging than conventional cellular networks [7,8] and DF based MWRNs, as the relay node will forward useful signals as well as the noise to all the users. To the best of the authors' knowledge, closed-form results for the average sum-rate bounds of AF MWRNs with full data exchange have not been reported in the literature. In this letter, we first analyze the achievable sum-rate lower bound and upper bound for the AF MWRN with full-data exchange, where a half-duplex relay helps K single-antenna users to receive the information from each other. Based on which, a closed-form expression for the average sum-rate lower bound is derived for the symmetric AF MWRN with a large number of users. Analytical results show that the relay power budget plays an important role on the achievable sum-rate. Numerical results are presented to verify the analysis and it is shown that the derived bounds are tight.

2. System model

We consider a MWRN with K users and a single relay node. Each node is equipped with a single antenna and operates in the half-duplex mode. As in [5,6], we assume the K users exchange information via the relay. We consider full-data exchange, i.e., each user decodes the messages from all the other users in two consecutive phases. In the first phase (referred to as MAC phase), all the K users transmit to the relay node. The relay broadcasts signal to the users in the second phase (referred to as BC phase). The channel from user k to the relay is denoted by h_k. We consider a symmetric time-division duplex (TDD)

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MWRN that the channels are independent and identically distributed (i.i.d.) frequency-flat Rayleigh fading with an average gain of G, i.e., $E(|h_k|^2) = G, \forall\, k$, and keep constant over the two phases as assumed in [5,6,10,11]. Throughout of this paper, we assume perfect channel knowledge at the nodes. For practical systems, several existing channel estimation algorithms in the literature [13,14] could be used to estimate the channels after proper modification.

In the MAC phase, the transmit signal of the k-th user is given by $x_k(n) = \sqrt{P_U}s_k(n),\ n = 1, \ldots, N$, where N denotes the number of symbols, P_U denotes the power used for information transmission, and $s_k(n)$ denotes the unit-power information signal. The received signal at the relay is

$$y_R(n) = \sum_{k=1}^{K} \sqrt{P_U}h_k s_k(n) + z_R(n), \tag{1}$$

where $z_R(n) \sim \mathscr{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN). Here, we assumed perfect timing synchronization that all the K users' transmitted signals are received at the relay node simultaneously as in [5,10,11].

During the BC phase, the relay broadcasts an amplified version of the received signal in the MAC phase to all the users. The transmit signal at the relay can be expressed as

$$x_R(n) = \sqrt{\beta}y_R(n), \tag{2}$$

where β is an amplification factor to meet the power constraint P_R at the relay:

$$\beta = \frac{P_R}{\sum_{k=1}^{K}|h_k|^2P_U + \sigma^2}. \tag{3}$$

In the BC phase, each user is able to remove the self-interference, provided that global channel state information is available [5,10]. After removing the self-interference, user k obtains

$$\widetilde{y}_k(n) = \sum_{m=1, m\neq k}^{K} \sqrt{\beta P_U}h_k h_m s_m(n) + \sqrt{\beta}h_k z_R(n) + z_k(n), \tag{4}$$

where $z_k(n) \sim \mathscr{CN}(0, \sigma^2)$ is the AWGN at user k.

After removing the self-interference from the received signal, the signal model in (4) represents a multi-access channel with $K-1$ users. Hence, the achievable rate region for the half-duplex AF MWRN is the convex hull of

$$\bigcap_{k=1}^{K}\bigcap_{\mathscr{S}\in\mathscr{S}_k}\left\{(R_1, \ldots, R_K)|\sum_{m\in\mathscr{S}} R_m \leqslant R_{k,\mathscr{S}}\right\}, \tag{5}$$

where $\mathscr{S}_k = \{1, \ldots, k-1, k+1, \ldots, K\}$, and

$$R_{k,\mathscr{S}} = \mathscr{C}\left(\frac{\beta|h_k|^2\sum_{m\in\mathscr{S}}|h_m|^2P_U}{\beta|h_k|^2\sigma^2 + \sigma^2}\right), \tag{6}$$

with $\mathscr{C}(x) \triangleq \frac{1}{2}\log_2(1 + x)$.

3. Performance bounds

Recall that the signal model in (4) represents a multi-access channel with $K-1$ users, and there are K such multi-access channels in the AF MWRN. Sort the channels in an ascending order as $|h_{\pi(1)}| \leqslant \ldots \leqslant |h_{\pi(K)}|$, where $\{\pi(1), \pi(2), \ldots, \pi(K)\} = \{1, 2, \ldots, K\}$. For the AF MWRN, the achievable rates of users $\pi(2), \ldots, \pi(K)$, are bottlenecked by the worst-channel user $\pi(1)$, while the data rate of user $\pi(1)$ is bottlenecked by the second worst user $\pi(2)$. Base on these observations, we have the following result on the achievable sum-rate of the AF MWRN.

Theorem 1. *The achievable sum-rate of the AF MWRN is bounded by*

$$R_{\text{sum}}^{\text{LB}} \leqslant R_{\text{sum}} \leqslant R_{\text{sum}}^{\text{UB}}, \tag{7}$$

where

$$R_{\text{sum}}^{\text{LB}} = \mathscr{C}\left(\frac{\beta|h_{\pi(1)}|^2\sum_{m=1}^{K}|h_m|^2P_U}{\beta|h_{\pi(1)}|^2\sigma^2 + \sigma^2}\right), \tag{8}$$

and

$$R_{\text{sum}}^{\text{UB}} = R_{\pi(1),\mathscr{S}_{\pi(1)}} + R_{\pi(2),\{\pi(1)\}}. \tag{9}$$

In addition, the gap between the lower bound (8) and the upper bound (9) tends to be zero as $K \to \infty$.

Proof. Consider the lower bound first. We assume successive interference cancellation (SIC) is employed at each user with the same decoding order $\pi(1), \pi(2), \ldots, \pi(K)$. Consider decoding at the user $\pi(1)$ with the worst channel gain. From the rate region (5), the achievable sum-rate of the other $K-1$ users is bounded by

$$\sum_{m=2}^{K} R_{\pi(m)} \leqslant R_{\pi(1),\mathscr{S}_{\pi(1)}}. \tag{10}$$

□

Recall from (4) that the channel seen by user $\pi(1)$ (after self-interference cancellation) is a multiple-access channel with $K-1$ users. From the information theory, it is known that the sum-rate $R_{\pi(1),\mathscr{S}_{\pi(1)}}$ for users $\pi(2), \pi(3), \ldots, \pi(K)$ in (10) is indeed achievable using SIC, that is:

$$\sum_{m=2}^{K} R_{\pi(m)} = R_{\pi(1),\mathscr{S}_{\pi(1)}}. \tag{11}$$

For the worst-channel user $\pi(1)$, the achievable rate is bottlenecked by the second-worst-channel user $\pi(2)$. With the decoding ordered as $\pi(1), \pi(3), \ldots, \pi(K)$ at user $\pi(2)$, the achievable rate of user $\pi(1)$ is

$$R_{\pi(1)} = \mathscr{C}\left(\frac{\beta|h_{\pi(2)}|^2|h_{\pi(1)}|^2P_U}{\beta|h_{\pi(2)}|^2(\sum_{n=3}^{K}|h_{\pi(n)}|^2P_U + \sigma^2) + \sigma^2}\right). \tag{12}$$

From the above two equations, the achievable sum-rate of AF MWRN is given by

$$R_{\text{sum}} = R_{\pi(1),\mathscr{S}_{\pi(1)}} + R_{\pi(1)}. \tag{13}$$

Since $|h_{\pi(2)}| \geqslant |h_{\pi(1)}|$ and $|h_{\pi(2)}| \geqslant 0$, then $R_{\pi(1)}$ can be lower bounded by

$$R_{\pi(1)} \geqslant \mathscr{C}\left(\frac{\beta|h_{\pi(1)}|^4P_U}{\beta|h_{\pi(1)}|^2(\sum_{n=2}^{K}|h_{\pi(n)}|^2P_U + \sigma^2) + \sigma^2}\right). \tag{14}$$

Note that from (6), $R_{\pi(1),\mathscr{S}_{\pi(1)}}$ is given by

$$R_{\pi(1),\mathscr{S}_{\pi(1)}} = \mathscr{C}\left(\frac{\beta|h_{\pi(1)}|^2\sum_{m=2}^{K}|h_{\pi(m)}|^2P_U}{\beta|h_{\pi(1)}|^2\sigma^2 + \sigma^2}\right), \tag{15}$$

Substituting (15) and (14) into (13), we have

$$\begin{aligned} R_{\text{sum}} &= R_{\pi(1),\mathscr{S}_{\pi(1)}} + R_{\pi(1)} \\ &\geqslant \mathscr{C}\left(\frac{\beta\,|\,h_{\pi(1)}\,|^2\sum_{m=2}^{K}|\,h_{\pi(m)}\,|^2\,P_U}{\beta\,|\,h_{\pi(1)}\,|^2\,\sigma^2+\sigma^2}\right) \\ &+ \mathscr{C}\left(\frac{\beta\,|\,h_{\pi(1)}\,|^4\,P_U}{\beta\,|\,h_{\pi(1)}\,|^2\,(\sum_{n=2}^{K}|\,h_{\pi(n)}\,|^2\,P_U+\sigma^2)+\sigma^2}\right) \\ &= \mathscr{C}\left(\frac{\beta\,|\,h_{\pi(1)}\,|^2\sum_{m=1}^{K}|\,h_m\,|^2\,P_U}{\beta\,|\,h_{\pi(1)}\,|^2\,\sigma^2+\sigma^2}\right), \end{aligned} \tag{16}$$

which is exactly the result in (8).

Now consider the upper bound. From the rate region (5), the data rate of the worst-channel user $\pi(1)$ is upper bounded by

$$R_{\pi(1)} \leqslant R_{\pi(2),\{\pi(1)\}} = \mathscr{C}\left(\frac{\beta|h_{\pi(2)}|^2|h_{\pi(1)}|^2P_U}{\beta|h_{\pi(2)}|^2\sigma^2 + \sigma^2}\right). \tag{17}$$

Then from (10) and the above equation, we conclude that the sum-rate of the proposed scheme is upper bounded by (9).