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A comprehensive Auxiliary Functions of Generalized Scattering Matrix (AFGSM) method to determine bandgap characteristics of periodic structures

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ABSTRACT

A comprehensive Auxiliary Functions of Generalized Scattering Matrix (AFGSM) method is presented on the bandgap analysis of periodic structures encountered in many engineering applications. Proposed method accurately determines the bandgaps of periodic structures with symmetric and asymmetric unit cells. In order to test the feasibility of the proposed method, numerical results are compared with Eigenvalue method (EIV) and HFSS/CST frequency domain simulations. The validity and applicability of the presented method are demonstrated by analysing the bandgap characteristics of periodically dielectric loaded rectangular waveguides, photonic crystals, helix Slow Wave Structures (SWSs) of Traveling Wave Tubes (TWTs) and excellent agreements on the simulation results are obtained. Bandgap analysis and design of different periodic structure problems can be effectively achieved using the proposed AFGSM method.

1. Introduction

Many engineering applications such as periodically dielectric loaded rectangular waveguides, photonic crystals, helix Slow Wave Structures (SWSs) of Traveling Wave Tubes (TWTs) and other electromagnetic bandgap structures include periodic structures (PSs) due to their electromagnetic properties. These structures are used to obtain bandgaps in the desired frequency bands or decelerate the velocity of electromagnetic waves as SWS [1–4]. In addition to this, they are also used in devices as filters [5], splitters and bends, etc. [6].

PSs consist of sequential repetition of unit cells (UCs). Due to periodicity, the bandgap characteristics and all the required information on the interested periodic structure can be inferred from the UC analysis. Hence, methods on the analysis of the UC take an important place in available literature. In this context, several numerical methods such as Finite Difference Time Domain (FDTD), Finite Element Method (FEM) and Method of Moments (MoMs) are used to determine bandgaps of periodic structures. These methods are full wave electromagnetic methods and require great computation time. On the other hand, when the UC approach is employed together with Floquet periodic boundary condition, periodic structure problem is reduced to the solution of an eigenvalue equation as a semi-analytical method reported in [7–10]. Conventional methods based on UC analysis that require the solution of eigenvalue equation are called Eigenvalue methods (EIVs). Dispersion diagram and electromagnetic or photonic bandgaps of periodic structures can be obtained in case of fine frequency mesh while the EIV methods are used. However, solution of eigenvalue equation is not appropriate for the design of the UC to achieve desired characteristics of periodic structures due to the requirement of fine frequency scanning and optimization of multi-variable in design space. Even though semi-analytical EIV methods are faster than pure numerical full wave electromagnetic methods, EIV methods require large execution time considering practical design applications. For this reason, fast and efficient methods have to be developed for the analysis and design of periodic structures.

An alternative technique which is called Auxiliary Functions of Generalized Scattering Matrix (AFGSM) method has been proposed in [11–13] for the analysis of the bandgap response of periodic structures without solving an eigenvalue equation. The method given in [11–13] is limited to the analysis and design of periodic structures with only symmetric UCs. However, asymmetric unit cells are commonly used in many applications. In this paper, to overcome this limitation we reformulate the AFGSM method and propose a comprehensive AFGSM method to determine bandgap characteristics of periodic structures for both symmetric and asymmetric UCs. The comprehensive AFGSM method is based on the analysis of stored complex power within the

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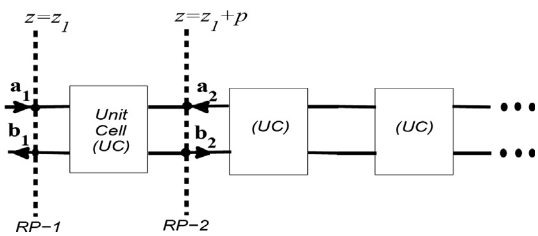

Fig. 1. GSM representation of a periodic structure.

unit cell of the periodic structure. To the best of our knowledge, the proposed formulation, which is not restricted to symmetric UCs, is original and has so far not been derived in the open literature. Introduced method proposes auxiliary functions (J_{+1} and J_{-1}) that determine band edge frequencies of the periodic structure by using Generalized Scattering Matrix (GSM) of the unit cell of the PS. The validity and applicability of the proposed comprehensive AFGSM method are investigated on the analysis of bandgap characteristics of lossless periodic structures which are periodically dielectric loaded rectangular waveguides, photonic crystals and helix SWSs of TWTs. Additionally, the considered problems are modeled and simulated with Ansys HFSS or CST full wave electromagnetic simulators to verify acquired results. The proposed original auxiliary functions (J_{+1} and J_{-1}) can easily and quickly determine bandgap characteristics of periodic structures and can be used to estimate stopband behavior of different types of periodic structures.

2. Theory

Unit cell of any periodic structure can be characterized with closed block and modeled with GSM representations as given in Fig. 1. GSM of unit cell is expressed as

$$\begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix} \tag{1}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are complex amplitudes of incident and reflected waves from the reference planes, respectively as shown in Fig. 1. Reference planes at $z = z_1$ ($RP-1$) and $z = z_1 + p$ ($RP-2$) are assumed as input and output ports on the definition of the elements of $\boldsymbol{S}$-matrix ($\boldsymbol{S}_{11}$, $\boldsymbol{S}_{12}$, $\boldsymbol{S}_{21}$, $\boldsymbol{S}_{22}$). The UC is periodic along the z-direction with a period of p. Due to periodicity of the UC, Floquet periodic boundary condition can be applied as

$$\boldsymbol{b}_2 = \lambda \boldsymbol{a}_1,\ \boldsymbol{a}_2 = \lambda \boldsymbol{b}_1 \tag{2}$$

Using Eqs. (1) and (2), we obtain the classical eigenvalue equation (EIV) for periodic UC as

$$\begin{bmatrix} \boldsymbol{I} & -\boldsymbol{S}_{11} \\ \boldsymbol{0} & -\boldsymbol{S}_{12} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{a}_1 \end{bmatrix} + \lambda \begin{bmatrix} -\boldsymbol{S}_{12} & \boldsymbol{0} \\ -\boldsymbol{S}_{22} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{a}_1 \end{bmatrix} = 0 \tag{3}$$

where $\boldsymbol{I}$ is the unit matrix. For single mode propogation, eigenvalue is defined as

$$\lambda_{1,2} = e^{\pm j\theta},\ \theta = \beta p \in (0, \pi) \tag{4}$$

where β is Floquet phase factor of the UC.

Dispersion diagram and passband-stopbands are determined by solving Eq. (3). As an alternative method, we know that observing the zero transitions of stored complex power ($\boldsymbol{\Psi}$) can be used to determine exact locations of passband and stopband edges. The stored complex power ($\boldsymbol{\Psi}$) expression can be written as [11]

$$\boldsymbol{\Psi} = \boldsymbol{a}^t \boldsymbol{Q} \boldsymbol{a}^* = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix}^t \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix}^* \tag{5}$$

with

$$\boldsymbol{Q}_{11} = (\boldsymbol{I} + \boldsymbol{S}_{11})\boldsymbol{U}(\boldsymbol{I} - \boldsymbol{S}_{11}^*) - \boldsymbol{S}_{12}\boldsymbol{U}\boldsymbol{S}_{12}^* \tag{6a}$$

$$\boldsymbol{Q}_{12} = \boldsymbol{S}_{12}\boldsymbol{U}(\boldsymbol{I} - \boldsymbol{S}_{22}^*) - (\boldsymbol{I} + \boldsymbol{S}_{11})\boldsymbol{U}\boldsymbol{S}_{12}^* \tag{6b}$$

$$\boldsymbol{Q}_{21} = \boldsymbol{S}_{12}\boldsymbol{U}(\boldsymbol{I} - \boldsymbol{S}_{11}^*) - (\boldsymbol{I} + \boldsymbol{S}_{22})\boldsymbol{U}\boldsymbol{S}_{12}^* \tag{6c}$$

$$\boldsymbol{Q}_{22} = (\boldsymbol{I} + \boldsymbol{S}_{22})\boldsymbol{U}(\boldsymbol{I} - \boldsymbol{S}_{22}^*) - \boldsymbol{S}_{12}\boldsymbol{U}\boldsymbol{S}_{12}^* \tag{6d}$$

where $\boldsymbol{U}$ denotes an $N \times N$ diagonal matrix and its elements are equal to the 1 or $\pm j$ for propagating and cutoff modes, respectively. Employing Eqs. (1) and (2), it can be written that $\boldsymbol{a}_2 = \boldsymbol{K}\boldsymbol{a}_1$ where $\boldsymbol{K} = (\boldsymbol{S}_{22}^{-1}(\lambda - \boldsymbol{S}_{21}))$. Hence the stored complex power $\boldsymbol{\Psi}$ can be expanded as

$$\boldsymbol{\Psi} = \boldsymbol{a}_1^t \boldsymbol{\Omega} \boldsymbol{a}_1^*, \quad \boldsymbol{\Omega} = (\boldsymbol{Q}_{11} + \boldsymbol{Q}_{12}\boldsymbol{K}^* + \boldsymbol{K}^t\boldsymbol{Q}_{21} + \boldsymbol{K}^t\boldsymbol{Q}_{22}\boldsymbol{K}^*) \tag{7}$$

In Ref. [11], Eq. (7) is used under symmetric UC assumption so that to apply even and odd symmetric excitations in the solutions. Finally, complex power stored by even and odd modes are derived in Ref [11] and these solutions are used to determine bandedge frequencies of periodic structures with symmetric UCs. Therefore, the method given in [11] is limited for periodic structures with symmetric UCs. The analysis valid for periodic structures with symmetric/asymmetric UCs given in this paper will take away these limitations and special cases. Derivations of the proposed method will be obtained starting from Eq. (7).

Eq. (7) does not yield an advantage for solving eigenvalue equation in its present form. Yet, if we assume that one Floquet mode propagates along the UC then bandgaps occur at $\lambda = 1$ and $\lambda = -1$ values. When we substitute $\lambda = 1$ and $\lambda = -1$ in Eq. (2) and replace b_2 and a_2 into the Eq. (1), we obtain $a_2 = a_1(1 - S_{21})/S_{22}$ or $a_2 = a_1(-1 - S_{21})/S_{22}$ for $\lambda = 1$ and $\lambda = -1$ values. In this case, Eq. (7) can be simplified as given in Eq. (8)

$$\Psi_{\pm 1} = |a_1|^2 (Q_{11} + Q_{12}K_{\pm 1}^* + Q_{21}K_{\pm 1} + Q_{22}|K_{\pm 1}|^2) \tag{8}$$

where

$$K_1 = \frac{1 - S_{21}}{S_{22}}, \quad K_{-1} = \frac{-1 - S_{21}}{S_{22}} \tag{9a}$$

$$Q_{11} = (1 + S_{11})(1 - S_{11}^*) - S_{12}S_{12}^* \tag{9b}$$

$$Q_{12} = S_{12}(1 - S_{22}^*) - (1 + S_{11})S_{12}^* \tag{9c}$$

$$Q_{21} = S_{12}(1 - S_{11}^*) - (1 + S_{22})S_{12}^* \tag{9d}$$

$$Q_{22} = (1 + S_{22})(1 - S_{22}^*) - S_{12}S_{12}^* \tag{9e}$$

For lossless periodic structure, real part of complex power satisfies $Re\{\Psi\} = 0$ condition in both passband and stopband. Furthermore, at the band-edge frequencies, imaginary part of complex power satisfies condition of $Im\{\Psi_{\pm 1}\} = 0$ for $\lambda = \pm 1$. Therefore, the functions of J_1 and J_{-1} given in below have to pass through zero precisely at the band edge frequencies due to $Im\{\Psi_{\pm 1}\} = 0$.

$$J_1 = Im\{Q_{11} + Q_{12}K_1^* + Q_{21}K_1 + Q_{22}|K_1|^2\} = 0 \tag{10}$$

$$J_{-1} = Im\{Q_{11} + Q_{12}K_{-1}^* + Q_{21}K_{-1} + Q_{22}|K_{-1}|^2\} = 0 \tag{11}$$

For lossless and reciprocal periodic structure, Q parameters reduce to $Q_{11} = j2Im(S_{11})$, $Q_{12} = Q_{21} = j2Im(S_{21})$, $Q_{22} = j2Im(S_{22})$ and auxiliary functions (J_1, J_{-1}) can be obtained as given in Eqs. (12) and (13)

$$J_1 = 2Im\{S_{11} + 2S_{21}Re\{K_1\} + S_{22}|K_1|^2\} = 0 \tag{12}$$

$$J_{-1} = 2Im\{S_{11} + 2S_{21}Re\{K_{-1}\} + S_{22}|K_{-1}|^2\} = 0 \tag{13}$$

As a result, bandgaps of periodic structures can be accurately obtained by observing zero transitions of J_1 and J_{-1}. It should be noted that auxiliary functions of J_1 and J_{-1} are original and have so far not been derived in available literature. These auxiliary functions are also convenient to solve many different types of periodic structure problems such as periodically loaded waveguides, traveling wave tubes and photonic crystals. Hence, the proposed method will be called as comprehensive AFGSM method. It should be noted that the proposed method can also be used in multi Floquet mode region with an acceptable accuracy. In the following section, we prove effectiveness of the