Int. J. Electron. Commun. (AEÜ) 93 (2018) 19–25

Contents lists available at ScienceDirect

Int. J. Electron. Commun. (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Energetically consistent modeling of passive memelements

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ARTICLE INFO

2010 MSC:
00-01
99-00

Keywords:
Memristive devices
Memgyrator
Memtransformer
Memcapacitor
Meminductor

ABSTRACT

Memory elements are suited for building self-organizing circuits. In contrast to memristive devices (nonlinear resistors with memory), memreactive devices (nonlinear capacitors or inductors with memory) are lossless and can be utilized in order to achieve more degrees of freedom in electrical circuits with respect to adaptively adjustable parameters.

Fabrication of pure memreactive elements is hard yet and hence mathematical models are needed to make pioneering pre-investigations. Modeling of memreactive elements is closely related to the modeling of nonlinear reactive elements. Common memreactive models are based on a static definition. Thus, they are not passive in general. But losslessness of such devices is of great importance from a circuit theoretic point of view. We propose a modeling approach for lossless memreactive elements based on an energetic definition. In this context, a novel equivalent circuit of memory devices including memtransformer or memgyrator is introduced. A known meminductive model is utilized in simulations and comparisons between different definitions are shown in order to underly the necessity of an energetically consistent model.

1. Introduction

Electrical circuits with adaptively adjustable parameters are used in order to preserve a desired functionality under variable environmental conditions. Adaptive filters, as general signal processing blocks, are popular examples of such systems. Whenever a filter's parameters must track time-varying conditions, adaptive filters are attractive options. In particular, in high-speed applications adaptive analog filters are more important than their digital equivalents. These filters mostly are realized by exploiting an error minimizing feedback-loop [1]. The adaption in such systems appears in an instantaneous regulation of the error, where the desired behavior should be known a priori. We intend to use adjustable elementary circuit elements with memory in order to achieve a similar functionality in a more efficient manner. An innovative example of such an element is the memristor, also referred to as memristive device or system, which is essentially a nonlinear resistor with memory [2]. The memory effect of these devices makes them suitable for an adaption of circuit parameters but in a self-organizing manner. This means, that the parameter adjustment is associated to a kind of a learning behavior depending on environmental experiences done in the past. With this, the adaption is more efficient and is not only related to an instantaneous reaction to environmental changes. Popular examples, where these elements have been used are neuromorphic circuits [3,4]. There, the memristive devices replace synapses in order to interconnect neurons. A more general approach is the interconnection of oscillators through memristors [5,6]. Moreover, an adaptively adjustable oscillator circuit including memristors and capable of anticipation of general information has been presented in [7,8].

Memristors are elements with losses and thus they belong to the class of dissipative systems. Considering electrical filters or oscillators, memristors can be utilized to adapt resistive parameters. Hence, the quality factor of an oscillator, for example, can be adjusted with such elements, whereas the resonance frequency is only marginally influenced. In order to achieve more degrees of freedom, lossless reactive elements with memory – memcapacitances or meminductances – are preferable. Some applications of memreactive elements can already be found in the literature, e.g. in the context of memcomputing or in neural networks [9].

Unfortunately, the fabrication of pure memreactive elements is hard yet. Memreactive behavior has been observed mostly as parasitic effects in memristive devices [10–12]. Therefore, mathematical models of memreactive elements are even more important in order to make pre-investigations of circuits including them before fabrication. A lot of mathematical models of memreactive elements, namely memcapacitances as well as meminductancens, are available [13–16]. Another approach regarding memreactive models is about mixed signal circuits consisting of microcontroller and operational amplifier imitating memreactive behavior [17].

Common existing mathematical models are based on a static definition of nonlinear reactive one-ports and hence they are in general not

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https://doi.org/10.1016/j.aeue.2018.05.023
Received 15 May 2018; Accepted 20 May 2018
1434-8411/

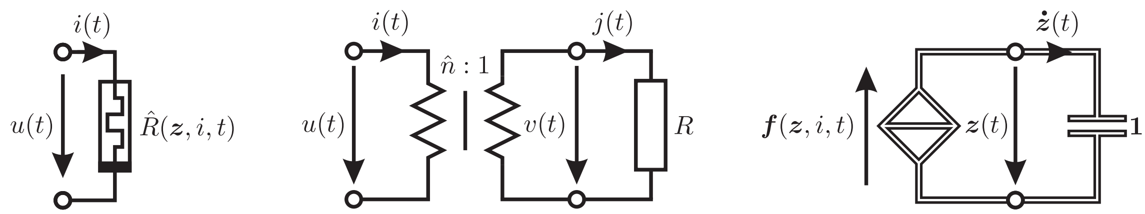

Fig. 1. Electrical representation of an *n*th-order *i*-controlled memristive system. Memristive one-port (left), equivalent circuit with memtransformer (middle), and electrical interpretation of the memory (right).

passive. Depending on incorporated parameters and the utilized excitation, an active behavior of such models can be observed [13], which is unfortunate from a circuit theoretic point of view. By definition, these elements should be even lossless. In particular, in the context of memristive or memreactive emulators an energetically consistent model is unavoidable [18–21]. We are intended to find an energetically consistent modeling approach for passive memristive and lossless memreactive elements independently of concrete model parameters and excitations. Although evaluating the passivity of memristive systems is a trivial task by inspecting the memristance, starting our proposed approach with these elements facilitates the accessibility of reactive devices with memory.

Modeling of memreactive elements is closely related to the modeling of nonlinear reactive elements. It is known that a static definition of nonlinear reactive elements leads to non-passive models, whereas an energetic definition yields energetically consistent models with respect to positive device values [22]. Those definitions depend on a suitable choice of complimentary constitutive variables. In accordance with the definition of general memristors, we define an *n*th-order *x*-controlled memory device by

$$y(t) = h(\boldsymbol{z}, x, t)x(t), \quad \dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}, x, t), \tag{1}$$

where the generalized response h relates the input and output signal x and y, respectively. The differential equation with the n-dimensional memory function $\boldsymbol{f}$ forms the memory of this device having an n-dimensional state vector $\boldsymbol{z}$. Depending on the device, x and y must be chosen appropriately such that they are complementary constitutive variables. This aspect will be consolidated in detail later on. We propose the adaption of the insights with respect to modeling nonlinear reactive elements to the modeling approach of memreactive elements. An energetic definition of memreactive elements will turn out in a parametric representation of the memory with energy-neutral two-ports, namely memtransformer and memgyrator [23,24]. First investigations of energetically consistent memcapacitive models can be found in [25]. Since the correct evaluation of the hysteresis has to be reformulated considering the proposed novel approach, a paradigm change in modeling of memreactive elements has been introduced with this paper.

The paper is organized as follows: In the next Section 2, the mathematical model of a general memristor is recapitulated. In this context, energy-neutral two-ports like memtransformer and memgyrator are introduced in order to get an alternative electrical interpretation of the memory effect. Afterwards, in Section 3 different modeling approaches of memreactive elements including energetic, static, and differential definitions are presented. Simulation results show the differences between those definitions and the importance of an energetically consistent model. Main results of the work are summarized in the conclusion at the end of the manuscript.

2. Memristor

In general, memristors are nonlinear resistors with memory. A state-dependent resistance value of such elements leads to adjustable resistive parameters in electrical circuits, which are in particular beneficial in self-organizing systems [3,7,8]. The corresponding dual elements are memductors. In the next paragraph, the mathematical definition of those elements is recapitulated. In this context, a novel electrical representation of such devices based on transformers and gyrators with memory is introduced.

2.1. Memristance

The *n*th-order *i*-controlled memristive system [2]

$$u(t) = \hat{R}(\boldsymbol{z}, i, t)i(t), \quad \dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}, i, t)i(t), \tag{2}$$

with memristance $\hat{R}$ as the general response and memory function $\boldsymbol{f}$, is a generic description of a memristor. The current i and voltage u are defined as the input and output variables, respectively. The appropriate choice of these variables plays a key role in modeling reactive devices with memory, as shown later in the manuscript. Memristors are dissipative systems except for the special cases where the memristor degenerates to a short-circuit or to an open-circuit, and hence it is passive for $\hat{R} \geqslant 0$. We intend to find a parametric representation of passive memristive systems that is as simple as possible. To this end, the memory effect of these devices are incorporated into the transmission ratio of an ideal transformer

$$n = \hat{n}(\boldsymbol{z}, i, t). \tag{3}$$

This leads to ideal transformers with memory – memtransformers. The memtransformer

$$u(t) = \hat{n}(\boldsymbol{z}, i, t)v(t), \quad j(t) = \hat{n}(\boldsymbol{z}, i, t)i(t), \tag{4}$$

where the voltage u and current i with respect to the primary side are interrelated to the voltage v and current j of the secondary side via the transformation ratio $\hat{n}$, is energy-neutral, independently of the transformation ratio, cf. Fig. 1 (middle). This can be verified from the instantaneous power transport of the two-port

$$p(t) = u(t)i(t) - v(t)j(t) = 0. \tag{5}$$

Describing the memristance in terms of the state-dependent transmission ratio

$$\hat{R}(\boldsymbol{z}, i, t) = \hat{n}^2(\boldsymbol{z}, i, t)R \tag{6}$$

results in the equivalent circuit of Fig. 1 (middle). Here, the memory effect of the system has been considered by an integrator circuit consisting of an ideal current source interconnected to a capacitor, cf. Fig. 1 (right). In general, such devices could have more than one internal state. Therefore, the integrator circuit is denoted as a multi-port. Since the memtransformer is energy-neutral independently of the transmission ratio and it is port-wise interconnected with a constant resistor R, the whole system is passive if and only if $R \geqslant 0$. In case of $\hat{n} = 0$, the primary side of the memtransformer degenerates to a short-circuit, whereas the secondary side becomes to an open-circuit and hence passivity is still guaranteed. Consequently, regarding the alternative electrical representation, it suffices to examine the value of the constant resistor in order to evaluate if a memristive system is passive or not. The benefits of such an electrical representation become even more obvious in case of reactive elements with memory.