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# Complex dynamics of a 4D Hopfield neural networks (HNNs) with a nonlinear synaptic weight: Coexistence of multiple attractors and remerging Feigenbaum trees

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ABSTRACT

This contribution investigates the nonlinear dynamics of a model of a 4D Hopfield neural networks (HNNs) with a nonlinear synaptic weight. The investigations show that the proposed HNNs model possesses three equilibrium points (the origin and two nonzero equilibrium points) which are always unstable for the set of synaptic weights matrix used to analyze the equilibria stability. Numerical simulations, carried out in terms of bifurcation diagrams, Lyapunov exponents graph, phase portraits and frequency spectra, are used to highlight the rich and complex phenomena exhibited by the model. These rich nonlinear dynamic behaviors include period doubling bifurcation, chaos, periodic window, antimonotonicity (i.e. concurrent creation and annihilation of periodic orbits) and coexistence of asymmetric self-excited attractors (e.g. coexistence of two and three disconnected periodic and chaotic attractors). Finally, PSpice simulations are used to confirm the results of the theoretical analysis.

## 1. Introduction

In biology, the human brain is the central organ of the nervous system. It controls most of activities of the body, processing, integrating, and coordinating the information it receives from the sensory organs. The brain dynamics is generally studied based on mathematical equations, usually obtained from artificial neural networks models [1–3]. Among these neural networks, Hopfield neural networks stand amongst the simplest paradigms. This is why in neurocomputing, Hopfield type neural network has an important use [4]. Since it is relatively simple, it can describe brain dynamics and provide a model for better understanding human activity and memory. In the brain dynamics, the signal generated is called electroencephalograms (EEGs) seems to have uncertain features, but there are some hidden samples in the signals [5]. Moreover, these signals are very sensitive to any changes in the brain's synaptic weights [6]. In this way, the brain dynamics is related to butterfly effect. It has been presented in the work of Panahi and collaborators [3] that, sometimes the neural activity in the brain can change from chaotic to periodic. The authors have also presented that, the chaotic behavior correspond to the normal state of the brain while the periodic state correspond to the pathological behavior of the brain named epileptic seizure.

In order to have more information on the brain's dynamics, several works have been carried out on the analyses of HNNs, and results have indicated some complex nonlinear phenomena such as chaos including single scroll and double scroll attractors, transient chaotic behaviors, hyperchaos, hidden attractors, just to name a few [3–13]. In particular, Bao and collaborators [13], remove the self-connection synaptic weight of the second neuron to simplify the HNNs connection topology presented in [9]. The investigations of Bao highlighted the complex phenomenon of the coexistence of attractors for the same set of synaptic weight matrix. Their results have been validated experimentally. It should be noted that, the phenomenon of the coexistence of attractors observed in HNNs have already been identified by Kengne and collaborators in several classes of nonlinear dynamical systems such as the Chua's system, Jerk systems, Duffing–Holmes type chaotic oscillators, memristor-based Shinriki's circuit and so on [14–21]. From these works, up to six disconnected solutions are captured in some Jerk systems as well as the phenomenon of antimonotonicity. It is then, of interest to see if a 4D HNNs with a nonlinear synaptic weight, modelling complex biological system such as brain dynamics is able to experience such type of nonlinear dynamics.

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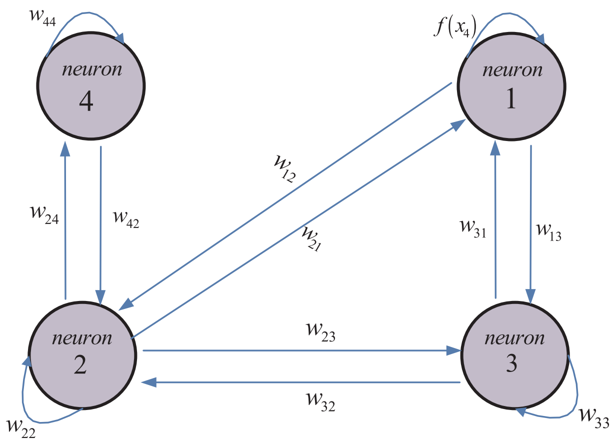


**Fig. 1.** Topological connection of a 4D Hopfield Neural Networks (HNNs).

Inspired by these previous works related to the dynamics of the Hopfield neural networks and complex nonlinear dynamical behaviors already found in some systems reported in the literature, we propose in this paper a mathematical model of a Hopfield neural networks obtained by introducing a nonlinear self-synaptic weight connection in the first neuron. The main objective of this paper is to show that the new Hopfield neural networks exhibits richer (e.g. antimonotonicity and coexistence of at least three asymmetric disconnected attractors) dynamics compared to previously published ones. Consequently, it gives an idea on the origin of epileptic fit which can be viewed as coexistence of normal and pathological state of the patient. Thus, the scientific contributions of this work can be summarized in the following lines:

(a) To carry out a systematic analysis of the proposed 4D HNNs and explain the chaos mechanism.
(b) To define the set of the synaptic weight matrix in which the model experience multiple coexisting attractors, hysteretic dynamics, and parallel bifurcations.
(c) To carry out Pspice simulations of the proposed HNNs to support the theoretical predictions.

The remaining parts of this scientific contribution are organized as follows. In Section 2, we propose and describe the novel 4D Hopfield neural network model. Analyses are carried out in terms of equilibria stability and existence of attractors. In Section 3, traditional nonlinear diagnostic tools such as bifurcation diagrams, graphs of Lyapunov exponents, phase portraits, and frequency spectra are exploited to highlight complex phenomena such as the coexistence of bifurcations, the coexistence of multiple asymmetric attractors and antimonotonicity. In Section 4, Pspice simulations are carried out to support our investigations. Finally, some concluding remarks and proposals for future works are given in Section 5.

## 2. 4D Hopfield neural networks with a nonlinear synaptic weight

### 2.1. 2.1. Mathematical expression of the proposed model

Hopfield neural networks (HNNs) are one among several artificial neural Networks which are generally used to describe brain dynamics. In such type of neuron, the circuit equation can be described as

$$C_i \frac{dx_i}{dt} = -\frac{x_i}{R_i} + \sum_{j=1}^{n} w_{ij} \tanh(x_j) + I_i \tag{1}$$

where $x_i$ is a state variable corresponding to the voltage across the capacitor $C_i$, $R_i$ is a resistor showing the membrane resistance between the inside and outside of the neuron, $I_i$ is the input bias current. The matrix $W = w_{ij}$ is a $n \times n$ synaptic weight matrix showing the strength of connections between the i-th and j-th neurons and $\tanh(x_j)$ is the neuron activation function indicating the voltage input from the j-th neuron. In neuroscience, synaptic weight refers to the strength or amplitude of a connection between two nodes, which corresponding in biology to the amount of influence the firing of one neuron has on another [7]. In this paper, we consider that, $C_i = 1$, $R_i = 1$, $I_i = 0$ and $n = 0$. When analog computer of the HNNs is computerized, the synaptic weight $w_{ij}$ neuron is a resistor. Based on these various hypotheses, we propose a 4DHopfield neural networks a nonlinear synaptic weight with connection topology presented in Fig. 1. From the general connection topology, the synaptic weight matrix is given as follow:

$$W = \begin{bmatrix} w'_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{24} \\ w_{31} & w_{32} & w_{33} & w_{34} \\ w_{41} & w_{42} & w_{43} & w_{44} \end{bmatrix} = \begin{bmatrix} f(x_4) & -0.84 & 0.5 & 0 \\ w_{21} & 1.5 & 0.9 & 0 \\ -5 & 0 & -0.1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2}$$

where $f(x_4) = w_{11}(a - b\tanh(x_4))$ with $a = 1$ and $b = 0.2$ represents the nonlinear synaptic weight of the model. From this nonlinearity, it is observed that, the synaptic weight $w'_{11}$ of the first neuron is affected by the intrinsic value of the fourth neuron. Thus, this configuration can enable the brain's dynamics to be more complex. From all the above consideration, the smooth nonlinear fourth order differential equations describing the proposed 4-neuronsHopfield neural networks can be taken in a dimensionless form as:

$$\begin{cases} \dot{x}_1 = -x_1 + w_{11}(a - b\tanh(x_4))\tanh(x_1) - 0.84\tanh(x_2) + 0.5\tanh(x_3) \\ \dot{x}_2 = -x_2 + w_{21}\tanh(x_1) + 1.5\tanh(x_2) + 0.9\tanh(x_3) \\ \dot{x}_3 = -x_3 - 5\tanh(x_1) - 0.1\tanh(x_3) \\ \dot{x}_4 = -x_4 + \tanh(x_2) \end{cases} \tag{3}$$

with respect to Eq. (3), there are two synaptic weights from which bifurcations can be investigated.

### 2.2. Basic dynamics of system

Face to the coordinate's transformation $(x_1, x_2, x_3, x_4) \to (-x_1, -x_2, -x_3, -x_4)$ the proposed HNN is variable then, it exhibits asymmetric attractors and the possibility of coexistence of attractors related to the symmetry of the model is to exclude. The volume of contraction rate of the introduced HNNs is given by:

$$\Lambda = -4 + w_{11}(1 - \tanh^2(\bar{x}_1))(a - b\tanh(\bar{x}_4)) + 1.5(1 - \tanh^2(\bar{x}_2)) \\ -0.1(1 - \tanh^2(\bar{x}_3)) \tag{4}$$

Since, $a = 1$, $b = 0.2$ and $-1 < \tanh(\bar{x}_i) < 1$ for all $x_i (i = 1, \ldots, 4)$ with an appropriate choice of synaptic weight $w_{11}$ our model can be dissipative thus, can support attractors. We recall that, the dissipation property of a system guarantees the presence of bounded global attractor and analytical indication of the global attractor in the phase space [22,23]. Also, the confinement of the chaotic trajectories of system (3) is justified. The confinement of the model of 4D-HNNs using the approach described in [24,25] is as follow:

$$V(x_1, x_2, x_3, x_4) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2 + \frac{1}{2}x_4^2 \tag{4}$$

Its corresponding time derivative is given as

$$\dot{V}(x_1, x_2, x_3, x_4) = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + x_4\dot{x}_4 \\ = -(x_1^2 + x_2^2 + x_3^2 + x_4^2) + \tanh(x_1)(w_{11}f'(x_4)x_1 + w_{21}x_2 - 5x_3) + \\ \tanh(x_2)(-0.84x_1 + 1.5x_2 + x_4) + \tanh(x_3)(0.5x_1 + 0.9x_2 - 0.1x_3) \tag{5}$$

Let us consider that

$$v(x_1, x_2, x_3, x_4) = \tanh(x_1)(w_{11}f'(x_4)x_1 + w_{21}x_2 - 5x_3) + \\ \tanh(x_2)(-0.84x_1 + 1.5x_2 + x_4) + \tanh(x_3)(0.5x_1 + 0.9x_2 - 0.1x_3) \tag{6}$$

Eq. (5) can then be rewritten as