Int. J. Electron. Commun. (AEÜ) 90 (2018) 110–122

Contents lists available at ScienceDirect

Int. J. Electron. Commun. (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Coexisting bifurcations in a memristive hyperchaotic oscillator

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ARTICLE INFO

Keywords:
Memristor
Coexisting attractors
Hyperchaos
Two parameter Lyapunov exponents
Experimental study

ABSTRACT

This paper investigates the dynamical behavior of the Tamasevicius et al. (1997) oscillator (named TCMNL hereafter) considering memristor as the nonlinear element by replacing the single diode in the original circuit. Various methods for detecting chaos/hyperchaos including bifurcation diagrams, spectrum of Lyapunov exponent, two parameter Lyapunov exponent, Poincaré sections and phase portraits are exploited to establish the connection between the system parameters and various complicated dynamics. By tuning the system parameters, some striking phenomena such as quasi-periodic oscillations and asymmetric pair of stable/unstable attractors are depicted. It is also found that the considered memristor induces the phenomenon of coexistence of attractors in wide ranges of bifurcation parameter. Finally, the hardware circuit is implemented and experimental observations are found to be in good agreement with the numerical investigations.

1. Introduction

Memristor (for MEMory ResISTOR) is the fourth fundamental circuit element in addition to resistors, inductors and capacitors. It was postulated in 1971 by Chua [2] and later implemented (as a nanoelectronic T_iO_2 device) in Hewlett–Packard (HP) laboratory by Stanley Williams and his team [3]. Conceptually, memristor is a two terminal nonlinear element with variable resistance called memristance which depends on the amount of electric charge that has passed through it in a given direction [4,5]. Clearly, memristors have the distinctive feature of memorizing the previous amount of electric charge that has passed through it and for how long the current has been applied. An extension of the notion of memristive systems [6] was later given by Chua and Kang, allowing that these systems depend additionally on a state. As a nonlinear element, memristor current-voltage (i–v) characteristic is a pinched hysteresis loop whose shape varies with frequency; As the goes to infinity, the shape tends to become linear [4,5]. This pinched characteristic is well known as one of the three fingerprints of memristor and also denotes the inherent nonlinearity of the device. The intrinsic nonlinearity of memristor is currently exploited for the design of novel chaotic circuits by replacing nonlinear resistance elements in classical chaotic/hyperchaotic circuits with memristor, thus, leading to rich repertoire of nonlinear behaviours [4,7–18]. Indeed, very recently, Ishaq et al. [13] reported both numerically and experimentally that the occurrence of chaotic beats in the driven Chua's circuit may be obtained by replacing the Chua's diode with an active flux controlled memristor. Using the same strategy, Ishaq and Lakshmanan [14] have observed the complex and rare phenomena of transient and persistent hyperchaos as well as hyperchaos beats in the memristive Murali-Lakshmanan-Chua (MLC) circuit. Not much later, same authors push their investigations and report the birth of a closed invariant curve known as Neimark-Sacker bifurcation as well as Hopf bifurcation on same memristive MLC circuit [15]. However, since HP memristors are currently commercially unavailable, all these investigations are based on memristors emulators. This has been a resurgence of research activities in developing new kinds of memristors or emulators in recent years to tackle all their inherent properties when considered in systems. Indeed, the pinched hysteresis loop of memristor is characterize through several nonlinearities including HP memristor model [19], cubic nonlinearity [17,20,21], non smooth piecewise nonlinearity [13,22] and smooth piecewise-quadratic nonlinearity [18,23,24] (just to name a few). In the meantime, practical circuit emulators are easily realized using off-the-

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https://doi.org/10.1016/j.aeue.2018.03.035
Received 14 February 2018; Accepted 28 March 2018
1434-8411/

shelf components such as resistors, capacitors, operational amplifiers, transistors and analog multipliers, which are particularly suited for breadboard experiments [25–29]. This is the case for memristive diode bridge-based circuits/emulator which was introduce by Bao et al. [25]. Apart from the simpler topological structure (consisting only of elementary electronic circuit components, and thus suitable for integrated circuits techniques), the memristive diode bridge emulator is also passive (*i.e* does not require any external/additional source of energy to exhibit its fingerprint) and henceforth should be preferred.

A lot of attention has recently been paid on multistable systems within which some experience infinitely many attractors [11,24] or exhibit hidden attractors [30–32]. Also, coexisting attractors have been reported in many nonlinear dynamical systems [16–18,33–40]. Such systems are of great importance in engineering applications as they can lead to unexpected and even catastrophic behavior in a system design. Many works reported that multiple attractors could be used as a source of unpredictability [41] (and references therein). Also, both theoretical investigations and engineering results showed that chaotic systems with multiple attractors are of great importance in communications [42]. The phenomenon of multistability has been examined in many memristor-based chaotic systems. For instance, Xu et al. [43], investigated this phenomenon in a nonideal active voltage-controlled memristor-based Chua's circuit. Very recently, Alombah et al. [44] reported the coexistence of four diverse attractors (*i.e* equilibrium point, a stable limit cycle, 16-peak periodic attractor and a strange attractor) in a charge-controlled memristive system. Recently, extreme multistability has also been reported in several memristor-based chaotic systems including hypogenetic jerk system [18], canonical Chua's circuit [16] and bistability in a third order Wien-bridge oscillator [17]. Also, Kengne et *al.* [45] used a generalized memristive system to obtain a new memristor based-chaotic system from Shinriki's circuit. Their investigations reveal the coexistence of asymmetric pair of periodic attractor with asymmetric pair of chaotic ones (*i.e* coexistence of four disconnected attractors by varying only initials states). Also, Kengne's group reveal the existence of multiple attractors in Jerk systems by considering both the memristor and antiparallel diodes as nonlinear elements [8,9,12]. In their works, authors exploited the symmetry of the studied system to derive a systematic method in the analysis of coexisting attractors. In this work, we follow the presented methods to report the coexistence of three disconnected attractors (asymmetric pair of stable/unstable attractors with symmetric stable one) in the voltage controlled memristive TCMNL oscillator.

Moreover, to the best of the present authors' knowledge, research on mutistability in hyperchaotic oscillators is still relatively less. In this work, a novel memristive TCMNL oscillator is obtained by replacing the nonlinear diode in the original circuit of [1] by a first order passive memristive diode bridge [25]. Distinguishing from most other hyperchaotic generators [46–50], TCMNL oscillator has attracted considerable attention, including synchronization using only one dynamical variable [51] for possible application in communication. Also in reference [52], Kengne reports the coexistence of chaos with hyperchaos as well as transient chaos both numerically and experimentally. On the other hand, the possible implementation of TCMNL oscillator using gyrators [53] instead of real inductors is well suited for audio communication purpose where several identical and stable inductance elements are also needed [1]. All these above mentioned works and their intriguing results mark the high interest devoted to the TCMNL oscillator and by the same way justify the attention paid on the analysis of memristive TCMNL hereafter in this paper. In fact, the above rich catalog of dynamical behaviors (*e.g* chaos, hyperchaos, transient chaos, bistability of chaos with hyperchaos just to name few) have already been reported in the literature. The novel memristive TCMNL oscillator exhibits the striking phenomenon of multiple attractors (*i.e* coexistence of three disconnected and distinct attractors) not previously reported (to the best of the authors' knowledge).

The layout of the paper is as follows. Section 2 introduces the circuit description of the memristive TCMNL oscillator following with the mathematical modeling process for further investigation on the dynamical behavior of the system. Also, basic properties and fixed points are also discussed. In Section 3, numerical analysis is performed. Combined bifurcation diagrams and corresponding graph of Lyapunov exponent spectrum are plotted to reveal possible scenarios to chaos/hyperchaos. The two parameter Lyapunov exponent tool is also used to show the degree of chaos/hyperchaos as well as the overall dynamics in the system when varying two control parameters. The phenomenon of coexisting attractors is discussed in Section 4 using bifurcation diagrams, Lyapunov exponent spectrum and basin of attractions as arguments. The experimental confirmation of all numerical investigations is reported in Section 5. Finally, in the last section, we present our conclusions and indicate possible further works.

2. Circuit description and state equations

2.1. Circuit description

The schematic diagram of Fig. 1(a) introduces the memristive TCMNL oscillator. The single semiconductor diode in the original circuit [1] is replaced by the memristor diode bridge of [25]. The memristor is considered as a nonlinear element [45,54] and is made of a full wave rectifier connected to a first order passive RC filter (see Fig. 1(b)). The memristive TCMNL is made of two harmonic oscillators (stable-L_2C_2 and unstable-L_1C_1) which are coupled through the passive diode-bridge memristor. The unstable harmonic oscillator consists of L_1C_1 loop in parallel with the negative impedance converter (NIC) [53]. The impedance of NIC ($Z_{NIC} = -R_1$) will serve as the main control parameter throughout our discussion. We would like to stress that the memristor is the only nonlinear component responsible for the hyperchaotic/chaotic and multistability behaviors in the complete system.

2.2. Mathematical model

We consider the generalized memristor consisting of a diode bridge with a first order parallel *RC* filter. Its mathematical representation [25] is expressed by the following equations:

$$i = g(V_{C_m},v)v = 2I_s exp(-\kappa V_{C_m})\sinh(\kappa v) \tag{1}$$

$$C_m\frac{dV_{C_m}}{dt} = f(V_{C_m},v) = 2I_s exp(-\kappa V_{C_m})\cosh(\kappa v) - \frac{V_{C_m}}{R_m} - 2I_s \tag{2}$$

where $\kappa = 1/2nV_T$ and I_s,n,V_T denote the reverse current, the emission coefficient and the thermal voltage of the diode, respectively [50,8,55]. i and v (see Eqs. (1) and (2)) represent the input voltage and current of the generalized memristor while V_{C_m} is the voltage across the capacitor C_m. The nonlinear functions $f(.)$ and $g(.)$ are describing the nonlinear characteristics of the memristor with $G_M = g(V_{C_m},v)$ representing the memductance. The following nominal values of circuit components are used to realize the memristive diode bridge: $C_m = 1\ \mu F, R_m = 1\ k\Omega$ and four 1N4148 diodes whose intrinsic parameters are $I_S = 2.682\ nA, n = 1.9$ and $V_T = 26\ mV$. It is worth mentioning that according to the work of Bao et al. [25], the diode bridge memristor exhibits the three fingerprints for identifying memristor.

Let us denote by I_{L_p} ($p = 1,2$) the current flowing through inductors (L_p,r_p) where r_p is the internal resistance of inductor L_p and by V_{C_j} ($j = 1,2$) the voltage across the capacitors C_j. Also, we assume capacitors, inductors and resistors are all linear in the circuit diagram of Fig. 1. By applying the Kirchhoff's law to the schematic diagram of Fig. 1, we obtained the following set of differential Eq. (3) describing the dynamics of the oscillator: