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A simple fractional-order chaotic system without equilibrium and its synchronization

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ABSTRACT

There has been an increasing interest in discovering no-equilibrium chaotic systems recently. In this paper, a novel three dimensional fractional-order chaotic system, which has no equilibrium, is introduced. Dynamics of the system has been studied. It is interesting that the system can exhibit coexisting chaotic attractors for the order as low as 2.7. The adjustable feature of a variable is studied by introducing a single controlled constant. Circuit implementation of the system is proposed to show its feasibility. In addition, we have designed the controllers to investigate coexisting synchronization types of such a new fractional-order system. Numerical examples have verified the proposed synchronization schemes.

1. Introduction

Fractional calculus has been studied and applied in different areas such as physics, electronics, mechanics, and control etc. [1–4]. In addition, applications of fractional systems in different domains of signal processing have been reported [5–10]. When considering the effects of fractional derivatives on chaotic systems, various fractional-order chaotic systems have been discovered [11–13]. Investigating new fractional-order chaotic systems is still an attractive topic because of their potential engineering applications range from secure communication, cryptography to electrically coupled neuron systems [14–16].

Recently, chaotic systems without equilibrium are attracting increasing interest due to the absence of equilibrium [17–19]. A striking feature of such no-equilibrium systems is the presence of "hidden attractor" [20]. For a hidden attractor, there is no intersection of a basin of attraction with small neighborhoods of equilibria [20]. We cannot localize hidden attractors numerically by a standard computational procedure [20,21]. Although numerous fractional-order chaotic systems with countable equilibrium points have been reported, there are few ones without equilibrium in the literature [22]. In addition to discovering of dynamics of such no-equilibrium systems, authors also paid attention on their synchronization. Li et al. found the first 4-D non-equilibrium fractional-order chaotic system [22]. By building a one-way coupling configuration, Li et al. obtained complete synchronization. Zhou and Huang constructed another 4-D fractional-order system without equilibrium point which is non chaotic for its integer-order but chaotic for its fractional-order as low as 3.2 [23]. In addition, a hyperchaotic fractional-order system without equilibria was derived from an integer-order hyperchaotic system [24]. Authors achieved hyperchaos synchronization via the observer-based method. A 4-D fractional-order no equilibrium cubic nonlinear resistor system was implemented in FPGA by Rajagopal et al. [25]. Rajagopal et al. developed the adaptive fractional-order sliding mode synchronization. Petráš extended a 3-D autonomous chaotic system without equilibrium to its fractional-order version [26]. Interestingly, Cafagna and Grassi presented a 3-D fractional system without equilibrium points which is chaotic for its fractional-order as low as 2.94 [27,28]. In order to determine chaos synchronization between two 3-D fractional-order systems, Cafagna and Grassi exploited observer-based method. Summarization of published fractional-order chaotic systems without equilibrium and their noticeable features are reported in Table 1. As can be seen in Table 1, researches have tended to focus on 4-D systems rather than 3-D ones. An additional problem is that there has been little discussion on synchronization of systems with different orders. Therefore, some questions that need to be raised are, for instance, if there exist other 3-D fractional-order chaotic systems without equilibrium or if it is possible to synchronize a 3-D fractional-order chaotic system without equilibrium and a 4-D fractional-order no-equilibrium one.

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Table 1
Published fractional-order chaotic systems without equilibrium: their typical features and synchronization schemes.

System	Dimension	Fractional order	Synchronization scheme
[25]	4	3.984	Adaptive sliding mode
[24]	4	3.84	Observer-based method
[22]	4	3.28	One-way coupling
[23]	4	3.2	No
[26]	3	2.94	No
[27,28]	3	2.94	Observer-based method
This work	3	2.7	Coexisting synchronization

The aim of this study is to examine a new 3D fractional system without equilibrium and its synchronization. The model of the fractional system is described in the next section. Section 3 presents the dynamics of the fractional system. In Section 4, we introduce an electronic circuit of the system. Different types of synchronization are reported in Section 5. Finally, the conclusion remarks are drawn in the last section. Our new contributions are to discover a new 3-D fractional non-equilibrium system with coexisting chaotic attractors and to investigate coexisting synchronization types of such a new fractional-order system. In addition, the fractional-order system has an adjustable variable, which has received significant attention recently. Our non-equilibrium system is different from integer-order systems without equilibrium [17–19,29] because of its fractional order. Due to the presence of coexisting chaotic attractors and its adjustable variable, such a non-equilibrium system is also different from known fractional-order systems without equilibrium [27,28]. Moreover, our synchronization scheme between the new 3-D fractional-order system without equilibrium and a 4-D fractional-order one without equilibrium is not similar to previous works as illustrated in Table 1.

2. Model of the fractional-order chaotic system without equilibrium

Previous studies have presented different definitions of fractional-order derivative, however Grunwald–Letnikov, Riemann–Liouville and Caputo definitions are commonly used [1,30,31]. In this work, we utilized the Caputo definition, which is defined by

$$ {}_0D_t^q f(t) = \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q+1-m}} d\tau, \quad m-1 < q < m. \tag{1} $$

In Caputo definition (1), m is the first integer which is not less than q ($m = \lceil q \rceil$) while Γ is the Gamma function:

$$ \Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt. \tag{2} $$

Recent evidence suggests that chaos can be observed in systems without equilibrium [17,18]. Different no-equilibrium chaotic systems has been reported in last years. Remarkably, a simple no-equilibrium system with six terms has been introduced in [32]. Because of its complex dynamics, such as chaos and multistability, this simple system is a typical example for studying systems without equilibrium. Derived from the integer-order system in [32], in this work we consider a fractional-order system described as follows:

$$ D^q x = y, \tag{3} $$

$$ \begin{aligned} D^q y &= -x-yz, \\ D^q z &= a\,|x| + xy-b, \end{aligned} $$

where three state variables are x,y,z while two positive parameters are a and b (a,$b > 0$). D^q denotes the Caputo fractional derivative with the initial time $t_0 = 0$. This form of Caputo fractional derivative is very helpful for getting the numerical solution of the fractional-order system. In system (3), the fractional order is denoted as q ($0 < q < 1$). As a result, the order of fractional system (3) is $3q$.

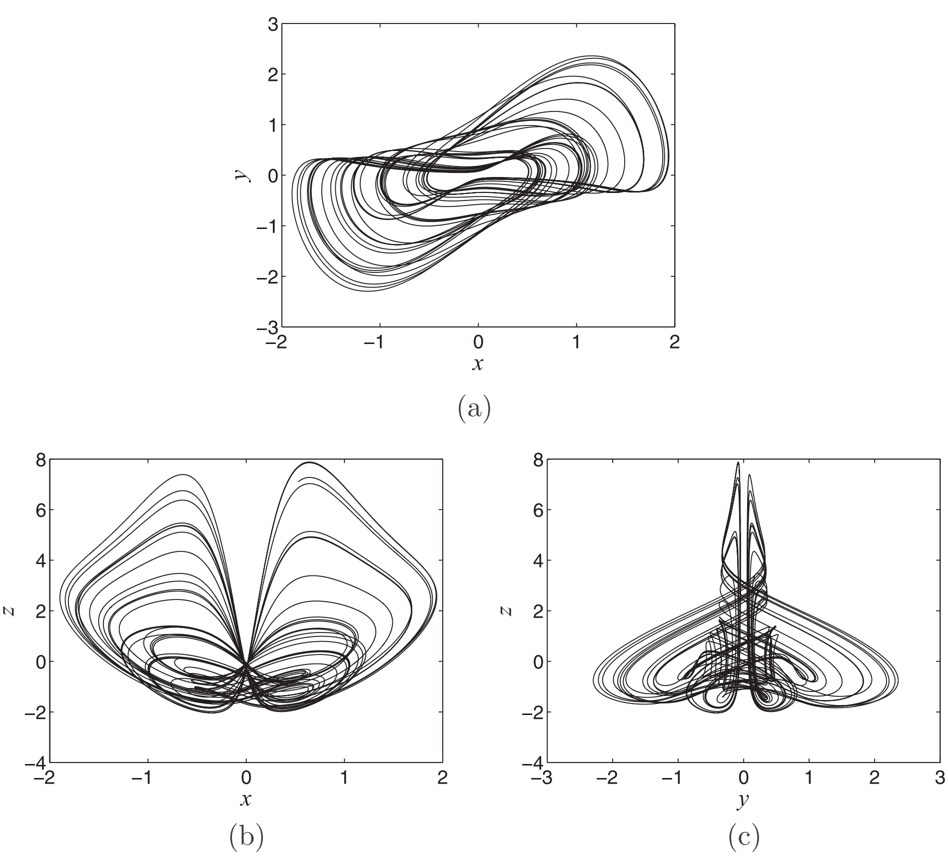

Fig. 1. Chaotic attractors in fractional-order system without equilibrium (3) for $q = 0.9$,$a = 2.5$,$b = 1.35$, and initial conditions $(x(0),y(0),z(0)) = (0,0.1,0)$: (a) x–y plane, (b) x–z plane, and (c) y–z plane.