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Particle filter for estimating multi-sensor systems using one- or two-step delayed measurements

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ABSTRACT

In this paper, we focus on designing a particle filter for a class of nonlinear discrete-time stochastic systems, where the multi-sensor measurements can be randomly and asynchronously delayed by one- or two- sampling periods. Under the independence assumption of multi-sensor delays, asynchronous delay model is built by using a separate set of Bernoulli random variables to describe the relationship between the ideal measurement and the actual measurement for each sensor. Based on the model, a new weighting scheme for particles is derived with the measurement delay fully considered. By incorporating the weighting scheme into the particle filtering framework, we obtain a new filter for systems with delayed measurements. The performance of the proposed filter is demonstrated by two numerical examples.

1. Introduction

In recent years, nonlinear filtering has been an active research field and plays an important role in many applications (see, e.g., [1–4] and the references therein). The classical filtering methods are based on the assumption that the measurements are available in a real-time manner. However, in many actual applications such as the communication networks with remote sensors, the received measurements may undergo random delay due to the network congestion. Hence, developing new nonlinear filters is urgently demanded to tackle the problem of Random Measurement Delay (RMD) [5].

Generally speaking, the filtering problems for systems with measurement delay can be divided into two categories according to whether the measurement data packets are time-stamped or not. For time-stamped cases, it is exactly known when the current received measurement is collected by a sensor, and many filters have been developed in Kalman filtering framework [6,7] or particle filtering framework [8,9].

For no time stamp cases, the measurement delay is considered to randomly arise with a certain probability and usually described by a set of stochastic parameters obeying Bernoulli distribution. Following this idea, in [10,11], a modified extended Kalman filter and a modified unscented Kalman filter for nonlinear systems with one- or Two-step RMD (TRMD) have been proposed based on the first-order linearization and unscented transformation respectively. Via Gaussian approximation of the one-step posterior predictive Probability Density Functions (PDFs) of state and delayed measurement, a Modified Cubature Kalman Filter (MCKF) [12] and the corresponding smoother [5] were proposed for nonlinear systems with One-step RMD (ORMD).

These estimates were developed in Gaussian approximation framework, where the distributions of state and measurement are assumed to be Gaussian. However, due to the nonlinear propagation of state, these distributions may be far away from Gaussian, which leads to degraded performance [13].

Recently, Particle Filters (PFs) [14] have proven to be promising alternatives to Gaussian approximation filters since they do not have the limitation imposed by the Gaussian assumption, and can yield optimal results asymptotically in the number of particles [2,15]. Recently, taking into account the multi-step RMD, a modified PF was also proposed in [16]. The derivation of the filter is based on an additional assumption, i.e., conditioned on the current state, the current received actual measurement is independent of the previous ones. Indeed, this assumption is true for no delay systems. However, it does not hold in the presence of measurement delay, which brings a theoretical problem to the algorithm. In

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addition, the filter is only suitable for single sensor systems or multi-sensor systems but with synchronous delay. In fact, multi-sensor asynchronous delay is more common in practice since measurements from different sensors may take different transmission links and consequently have different delay steps. In this paper, a more elaborate PF is developed to address the problem of RMD with delay steps no more than 2. The filter no longer suffers from the theoretical problems in [16] and can be used for systems with multi-sensor asynchronous delay.

2. Problem formulation

Consider the following multi-sensor nonlinear dynamic system

$$\boldsymbol{x}_t = \boldsymbol{f}_t(\boldsymbol{x}_{t-1}) + \boldsymbol{w}_{t-1} \tag{1}$$

$$\boldsymbol{z}_{t,k} = \boldsymbol{h}_{t,k}(\boldsymbol{x}_t) + \boldsymbol{v}_{t,k}, \quad k = 1, 2, \ldots, K \tag{2}$$

where $\boldsymbol{x}_t$ is the n-dimensional state vector with known initial distribution $p(\boldsymbol{x}_0)$, $\boldsymbol{z}_{t,k}$ is the m_k-dimensional ideal measurement collected by the k-th sensor at time t, $\boldsymbol{w}_t \sim p_w(\cdot)$ and $\boldsymbol{v}_{t,k} \sim p_{v,k}(\cdot)$ denote the white process noise and measurement noise respectively. The system function $\boldsymbol{f}_t(\cdot)$ and measurement function $\boldsymbol{h}_{t,k}(\cdot)$, as well as the PDFs $p_w(\cdot)$ and $p_{v,k}(\cdot)$, are known a priori. Define $\boldsymbol{z}_t$ to be the measurement set of the K sensors, i.e., let $\boldsymbol{z}_t = [\boldsymbol{z}_{t,1}, \boldsymbol{z}_{t,2}, \ldots, \boldsymbol{z}_{t,K}]^T$.

The multi-sensor asynchronous RMD model with the maximum possible delay step being 2 can be formulated as

$$\boldsymbol{y}_{t,k} = \begin{cases} \boldsymbol{z}_{t,k}, & t = 1, 2 \\ \sum_{d=0}^{2} \gamma_{t,k}^d \boldsymbol{z}_{t-d,k}, & t > 2 \end{cases}, \quad k = 1, 2, \ldots, K \tag{3}$$

where $\boldsymbol{y}_{t,k}$ is the current received actual measurement collected by the k-th sensor, d is the delay step, $\gamma_{t,k}^d, d = 0, 1, 2$ are Bernoulli random variables taking the value of 1 if $\boldsymbol{y}_{t,k}$ is delayed by d sampling periods. Define $\boldsymbol{y}_t = [\boldsymbol{y}_{t,1}, \boldsymbol{y}_{t,2}, \ldots, \boldsymbol{y}_{t,K}]^T$ to be the actual measurement set of the K sensors.

Define $\gamma_{t,k} = [\gamma_{t,k}^0, \gamma_{t,k}^1, \gamma_{t,k}^2]^T$. For any given t and k, the state space of $\gamma_{t,k}$ consists of 3 column vectors $\{\gamma_{t,k}^d\}_{d=0}^2$. In $\gamma_{t,k}^d$, the $(d+1)$-th element $\gamma_{t,k}^d$ takes the value of 1, while others take 0. For example, $\gamma_{t,k}^1 = [0, 1, 0]^T$. Random vector $\gamma_{t,k}$ obeys the known discrete distribution

$$p(\gamma_{t,k} = \gamma_{t,k}^d) = p_{t,k}^d, \; d = 0, 1, 2 \tag{4}$$

where $p_{t,k}^d \geqslant 0$ and $\sum_{d=0}^2 p_{t,k}^d = 1$. The random event $\gamma_{t,k} = \gamma_{t,k}^d$ indicates that, for the k-th sensor, the current received measurement $\boldsymbol{y}_{t,k}$ is delayed by d sampling periods. It is assumed that $\gamma_{t_1,k}$ is independent of $\gamma_{t_2,k}$ when $t_1 \neq t_2$, and γ_{t,k_1} is independent of γ_{t,k_2} when $k_1 \neq k_2$. We further assume the mutual independence of $\boldsymbol{x}_0$, $\{\boldsymbol{w}_t, t \geqslant 0\}$, $\{\boldsymbol{v}_{t,k}, t > 0\}_{k=1}^K$ and $\{\gamma_{t,k}, t > 0\}_{k=1}^K$.

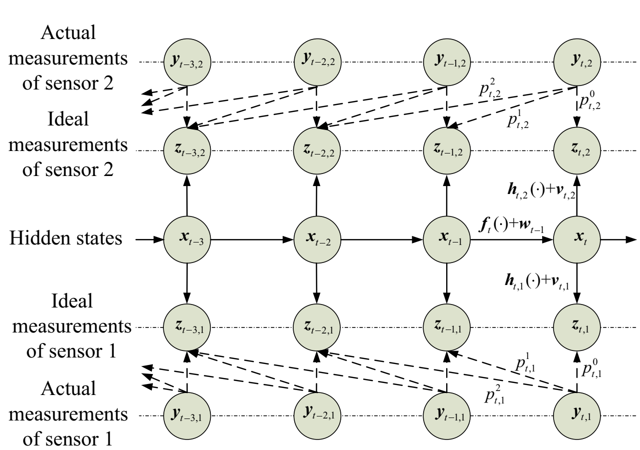

Fig. 1. Two-sensor asynchronous delay model with maximum possible delay step being 2.

For the case of $K = 2$, the dynamic system given by (1)–(3) can be illustrated by Fig. 1. Our aim is to recursively estimate $\boldsymbol{x}_t$ based on all available measurements $\boldsymbol{y}_{1:t} = \{\boldsymbol{y}_\tau\}_{\tau=1}^t$.

Remark 1. The ORMD model can be regarded as the special case of model (3). Indeed, if $p_{t,k}^2 \equiv 0$, for all $k = 1, 2, \ldots, K$, then model (3) reduces to ORMD model.

Remark 2. The RMD model given by (3) poses a great challenge to estimators due to the degraded quality of received measurements. Compared with $\boldsymbol{z}_{1:t}$, the actual measurement sequence $\boldsymbol{y}_{1:t}$ not only is out-of-sequence, but also suffers from measurement random missing. It is easy to prove that the ideal measurement $\boldsymbol{z}_{t,k}$, $t > 2$ is absent from the actual measurement sequence with probability $P_{miss} = (1 - p_{t,k}^0)(1 - p_{t+1,k}^1)(1 - p_{t+2,k}^2)$. Specially, if the delay probabilities are time-invariant, i.e. they can be represented by $p_{t,k}^0 = p_k^0$, $p_{t,k}^1 = p_k^1$ and $p_{t,k}^2 = p_k^2$, then P_{miss} reaches the maximum value 8/27 at the point $p_k^0 = p_k^1 = p_k^2 = 1/3$.

3. Particle filter for systems with multi-sensor TRMD

In PF, the joint PDF $p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t})$ is usually approximated by a set of weighted particle-trajectories as follows

$$p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t}) \approx \sum_{i=1}^N \pi_t^i \delta(\boldsymbol{x}_{0:t} - \boldsymbol{x}_{0:t}^i) \tag{5}$$

where $\delta(\cdot)$ is the Dirac delta function, $\boldsymbol{x}_{0:t}^i = \{\boldsymbol{x}_0^i, \boldsymbol{x}_1^i, \ldots, \boldsymbol{x}_t^i\}$ is the i-th particle-trajectory drawn from a pre-selected importance density function $q(\boldsymbol{x}_{0:t}|\boldsymbol{z}_{1:t})$ and assigned a normalized weight according to

$$\pi_t^i \propto \frac{p(\boldsymbol{x}_{0:t}^i|\boldsymbol{y}_{1:t})}{q(\boldsymbol{x}_{0:t}^i|\boldsymbol{y}_{1:t})} \tag{6}$$

Here, $\propto$ stands for 'proportional to'.

Assume that the weighted particle-trajectory set $\{\boldsymbol{x}_{0:t-1}^i, \pi_{t-1}^i\}_{i=1}^N$ is available. When $\boldsymbol{y}_t$ arrives, a new set $\{\boldsymbol{x}_{0:t}^i, \pi_t^i\}_{i=1}^N$ is required to approximate $p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t})$, where $\boldsymbol{x}_{0:t}^i \sim q(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t})$. Usually, $q(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t})$ is chosen to factorize such that

$$q(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t}) = q(\boldsymbol{x}_t|\boldsymbol{x}_{0:t-1}, \boldsymbol{y}_{1:t}) q(\boldsymbol{x}_{0:t-1}|\boldsymbol{y}_{1:t-1}) \tag{7}$$

Then $\boldsymbol{x}_{0:t}^i$ can be obtained by augmenting the existing particle-trajectory $\boldsymbol{x}_{0:t-1}^i \sim q(\boldsymbol{x}_{0:t-1}|\boldsymbol{y}_{1:t-1})$ with the new state $\boldsymbol{x}_t^i \sim q(\boldsymbol{x}_t|\boldsymbol{x}_{0:t-1}^i, \boldsymbol{y}_{1:t})$. By using the Bayes' rule, we have

$$\begin{aligned} p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t}) &= \frac{p(\boldsymbol{y}_t|\boldsymbol{x}_{0:t}, \boldsymbol{y}_{1:t-1}) p(\boldsymbol{x}_t|\boldsymbol{x}_{0:t-1}, \boldsymbol{y}_{1:t-1}) p(\boldsymbol{x}_{0:t-1}|\boldsymbol{y}_{1:t-1})}{p(\boldsymbol{y}_t|\boldsymbol{y}_{1:t-1})} \\ &\propto p(\boldsymbol{y}_t|\boldsymbol{x}_{0:t}, \boldsymbol{y}_{1:t-1}) p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}) p(\boldsymbol{x}_{0:t-1}|\boldsymbol{y}_{1:t-1}) \end{aligned} \tag{8}$$

If we choose $q(\boldsymbol{x}_t|\boldsymbol{x}_{0:t-1}, \boldsymbol{y}_{1:t}) = p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1})$, and perform resampling at each filter cycle just like the Standard PF (SPF) [14], then by substituting (7) and (8) into (6), the weight equation can be simplified as

$$\pi_t^i \propto p(\boldsymbol{y}_t|\boldsymbol{x}_{0:t}^i, \boldsymbol{y}_{1:t-1}) = p(\boldsymbol{y}_t|\boldsymbol{x}_{t-2:t}^i, \boldsymbol{y}_{1:t-1}) \tag{9}$$

The equality in (9) comes from the fact that, conditioned on $\boldsymbol{x}_{t-2:t}^i$, $\boldsymbol{y}_t$ does not depend on $\boldsymbol{x}_{0:t-3}^i$.

In [16,17], an additional assumption, i.e., $\boldsymbol{y}_t$ does not depend on $\boldsymbol{y}_{1:t-1}$, is made, which leads to a further simplified weight equation