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(1+α) Fractional-Order Transfer Functions to Approximate Low-Pass Magnitude Responses with Arbitrary Quality Factor

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Abstract

The coefficients of three fractional-order low-pass transfer functions are presented to aid in the design of these filters based on their arbitrary quality factors. These coefficients are found by minimizing the error between these fractional-order transfer functions and the second-order transfer function using numerical least squares optimization. Coefficients and design equations are presented for fractional-orders between one and two. Stability of the transfer functions with the presented coefficients are examined and possibilities of characteristic frequency shifting are shown. The results are verified by PSpice simulation of a conveyor-based low-pass filter with fractional order of 1.5 and quality factor $Q = 5$.

Keywords

Fractional-order filter, Analog filter, Fractional-order transfer function, Fractional calculus

1. Introduction

Fractional-order system theory is currently attracting the attention of researchers and specialists in many disciplines of science and engineering. These disciplines include signal processing [1], electrical engineering [2], [3], biology [4], and control theory [5] to name a few, though applications in many additional fields are being pursued [6]. This attention is due to the fact that the behavior of many systems (not only in those branches mentioned) can be modeled using fractional-order differential equations; containing derivatives of non-integer order over their integer-order counterparts. These fractional-order differential equations are from a generalization of calculus, known as fractional calculus which is the branch of mathematics that defines derivatives and integrals to non-integer orders [7] of which the traditional calculus is actually a smaller subset.

Applied to electrical engineering, the field of electronics is seeing the rapid import of concepts from fractional calculus to explore their impact and determine the possible advantages of designing systems using their properties. In the last decade applications of these concepts have been applied to the design of fractional-order analog filters [8] - [22], immittance circuits [23] -[25], oscillators [26] -[28], or controllers [29]. Recently, fractional-order systems have been shown to increase the design flexibility by increasing the number of degrees of freedom compared to the classical integer order cases. The arbitrary fractional order of circuits can be advantageously utilized, for example, i) to obtain constant and