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Polar codes with the unequal error protection property

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A R T I C L E I N F O

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A B S T R A C T

The Internet of Things (IoT) is expected to support a wide variety of devices with different transmission reliability requirements. Channel codes with the unequal error protection (UEP) property are rather appealing for such applications. As the first provably capacity-achieving codes, polar codes have excellent error-correcting performances over a wide range of block lengths with low encoding and decoding complexities, which makes these codes well-suited for the energy-constrained IoT applications. Polar codes with unequal error protection are desirable. However, the UEP property of polar codes has not been explored explicitly in existing works. In this paper, a construction method for UEP polar codes is proposed based upon a novel selection rule of bit-channels. Refreezing and reselecting operations are employed to enable the UEP property for polar codes. The upper and lower bounds on the block error rate (BLER) of each data block are derived. Simulation results show that the proposed codes can provide a good performance of unequal error protection.

1. Introduction

With rapidly increasing amounts of mobile devices and services, the fifth generation (5G) mobile communication systems have drawn dramatic interest all over the world, which aim to provide energy-efficient, low-latency and high-reliable communications. It is expected that the 5G system will be able to support three orders of magnitude higher capacity per km^2, a hundred times higher data rate, latency of less than 1 ms across the radio access link, a hundred times more connections (links) and three orders of magnitude lower energy consumption than the current generation of wireless networks [1].

The Internet of Things (IoT) is a crucial scenario of 5G systems, which is expected to support a variety of applications and services. Some emerging applications such as smart transportation, remote medicine and intelligent control will significantly change the way of our lives [2–4]. IoT is supposed to support a massive number of heterogeneous devices with limited energy. Different kinds of data with different reliability requirements are transmitted under these scenarios. Channel coding plays a significant role in supporting all the heterogeneous applications and services. Error control codes with the unequal error protection (UEP) property are desired under this condition. Unequal error protection codes were first studied in [5] and since then a lot of studies have been carried out. A variety of unequal error protection schemes have been designed for low-density parity-check (LDPC) codes [6–8], turbo codes [9], fountain codes [10–12], etc.

Polar codes proposed by Arikan are the first provably capacity-achieving codes of any given binary-input discrete memoryless channels (B-DMCs) [13]. Polar codes have been selected for 5G mobile communications systems [14]. Polar codes have low encoding/decoding complexity and explicit recursive structures, which contributes to low energy consumption at the terminal end. They have good error-correcting performances over a wide range of block lengths, especially short and medium code lengths. All these characters make polar codes well suited for the requirements of various IoT applications.

Polar codes take advantage of the channel polarization phenomenon. That is, following the operations of channel combining and channel splitting, the capacities of channels seen by every input bit, termed the bit-channel, become distinguished. Some of the bit-channels tend to be noiseless while some become almost purely noisy channels. The strategy of polar coding is to transmit information data through all the noiseless bit-channels at rate one, while transmitting fixed bits in the purely noisy bit-channels.

Therefore estimating the error probability of each bit-channel is crucial for constructing polar codes. Arikan proposed a recursive calculation algorithm based on the Bhattacharyya parameters for evaluating the reliability of the channel [13]. However, this method only has a recursive algorithm suited for the binary erasure channel (BEC). The Bhattacharyya parameter is an upper bound on the error probability with maximum likelihood (ML) detection, when transmitting a single bit. Arikan also applied Monte-Carlo simulations to evaluating the

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reliabilities of the bit-channels [13], which are of high complexity for large block lengths. Mori and Tanaka proposed the use of the density evolution (DE) tool to calculate the reliabilities of the bit-channels for arbitrary symmetric B-DMCs [15,16]. Density evolution involves convolution with an iteratively increasing data size, the complexity of which is not acceptable for large block lengths. A Gaussian approximation (GA) method is proposed in [17,18] to reduce the complexity of density evolution. The intermediate variables are seen as Gaussian variables. Thus only the mean and variance need to be estimated. Tal and Vardy introduced an approximation method based on degraded and upgraded channels in [19]. Instead of estimating the reliabilities of the original bit-channels, they introduced degraded and upgraded channels, which have higher or lower error probabilities than the original bit-channels, respectively. Through estimating the reliabilities of the degraded and upgrade channels, an upper and a lower bound on the error probabilities of the bit-channels are attainable.

Though polar codes offer the perfect performance under successive cancellation (SC) decoding when block lengths tend to infinity, the performance of polar codes with short or moderate block lengths are not satisfactory. Various kinds of powerful decoding algorithms with low or moderate decoding complexity have been designed for polar codes. Arikan proposed a belief propagation (BP) decoder for polar codes in [20]. The performance of BP decoding has improved compared to SC decoding, but there is still a performance gap between BP and maximum likelihood (ML) decoding. Tal and Vardy proposed the successive cancellation list (SCL) decoder for polar codes [21]. Simple concatenation with some cyclic redundancy check (CRC) bits is capable of substantially improving the performances of polar codes as well as outperforming ML decoding. The adaptive CRC-aided successive cancellation list (aCA-SCL) decoder for polar codes which iteratively increases the list size was proposed in [22]. The aCA-SCL decoder provides a significant complexity reduction in complexity as opposed to SCL decoding, and it is able to achieve a much better performance with an appropriate maximum list size. Several simplified or low-latency SCL decoders have been proposed [23,24], which makes polar codes better suit IoT applications.

Polar codes have also been shown to be able to achieve good performance over wide-ranging channels, e.g., wiretap channels [25–27], broadcasting channels [27,28] and multi-user channels [29,30]. The performance of polar codes in quantum channels has also been studied [26,31,32]. Moreover, polar codes are a perfect candidate for source coding [33–35]. Recently, studies have been carried out for capacity-achieving rate-compatible polar codes [36,37].

As a class of high-efficiency error-correcting codes, polar codes with the unequal error protection property are highly desirable. It is stated in [38] that the differences in the error probability for different bit-channels in the information set can be utilized to design UEP polar codes. In [39], the authors exploit the differences in the error probability of different bit-channels for image transmission. However, no explicit encoding scheme or theoretical analysis for UEP polar codes is given in both papers.

The major contributions of this paper are three-fold detailed as follows:

(1) We propose a new class of polar codes with the UEP property. We design a mapping rule for bit-channels for data with different reliability requirements. Refreezing and reselecting operations are introduced to improve the UEP performance;
(2) We investigate the performance of the proposed UEP polar codes. An upper and a lower bound on the block error rate (BLER) of each data block are derived. Specially, approximate BLERs are derived for more important bits (MIB) and less important bits (LIB) for the case of two importance levels; and
(3) We validate the efficiency of the proposed coding scheme via simulation results and elaborate on how the encoding parameters affect the performance of the proposed UEP polar codes.

The remainder of the paper is organized as follows. Section 2 presents a brief review of polar codes. In Section 3, an unequal error protection coding scheme for polar codes is proposed, followed by a performance analysis for the proposed UEP coding scheme in Section 4. Simulation results are presented in Section 5. Finally, concluding remarks are drawn in Section 6.

2. Preliminaries

2.1. Polar codes

Given a binary-input discrete memoryless channel (B-DMC) $W: \mathscr{X} \to \mathscr{Y}$, where $\mathscr{X} = \{0, 1\}$ and $\mathscr{Y}$ denote the input and output alphabets, respectively. The transition probabilities of W are $W(y|x)$, $x \in \mathscr{X}$, $y \in \mathscr{Y}$. Denoting by W^N be N independent uses of W, we have $W^N: \mathscr{X}^N \to \mathscr{Y}^N$ with

$$W^N(y_1^N|x_1^N) = \prod_{i=1}^{N} W(y_i|x_i). \tag{1}$$

These N independent channels are combined into one synthesized channel W_N with N binary inputs. The transition probabilities of W_N and W^N are related as follows

$$W_N(y_1^N|u_1^N) = W^N(y_1^N|u_1^N G_N). \tag{2}$$

The synthesized channel can be split into a set of binary-input coordinate channels, dubbed the bit-channels. Define the ith bit-channel $W_N^{(i)}: \mathscr{X} \to \mathscr{Y}^N \times \mathscr{X}^{i-1}$ with transition probabilities

$$W_N^{(i)}(y_1^N, u_1^{i-1}|u_i) \triangleq \sum_{u_{i+1}^N \in \mathscr{X}^{N-i}} \frac{1}{2^{N-1}} W_N(y_1^N|u_1^N). \tag{3}$$

The channel capacity of $W_N^{(i)}$ is distinct for $i = 1, 2, ..., N$. The essential idea behind polar coding is to choose the most reliable bit-channels to transmit information bits, while assigning fixed bits known to both the transmitter and receiver to those less reliable ones. Let N be the block length of polar code and $N = 2^n$ where n is a positive integer. Let $u_1^N = (u_1, u_2, ..., u_N)$ denote the input bit vector and $x_1^N = (x_1, x_2, ..., x_N)$ represent the output bit vector. Let $\mathscr{A}$ be a subset of indices of the most reliable k coordinate channels, where k is the code dimension. The input vector u_1^N is composed of two parts, i.e., the information bits $u_{\mathscr{A}}$ and the frozen bits $u_{\mathscr{A}^c}$ (say all zeros). The encoding process of polar codes can be described as:

$$x_1^N = u_1^N G_N, \tag{4}$$

where $G_N = B_N F^{\otimes n}$ is the generator matrix of order N, B_N is a bit-reversal permutation matrix, $F \triangleq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, and $\otimes n$ is the nth Kronecker power of F.

2.2. Construction of polar codes

The main task of constructing polar codes is to evaluate the error probability of each bit-channel and then choose an information set among the bit-channels so as to minimize the block error probability. We employ the Tal and Vardy construction method in this paper [19]. For continuous channels like the AWGN channel, a quantization process is first employed to transform the original channel to a binary-input memoryless symmetric (BMS) channel with a finite output alphabet size.

The transition probabilities of the bit-channels can be computed recursively using the channel transformations $W \boxplus W$ and $W \boxtimes W$ defined in [13]:

$$W \boxplus W(y_1, y_2|u_1) \triangleq \frac{1}{2} \sum_{u_2 \in \mathscr{X}} W(y_1|u_1 \oplus u_2) W(y_2|u_2), \tag{5}$$