Computer Communications 122 (2018) 20–29

Contents lists available at ScienceDirect

Computer Communications

journal homepage: www.elsevier.com/locate/comcom

Joint localization of multiple sources from incomplete noisy Euclidean distance matrix in wireless networks☆

Xiansheng Guo[a,b], Lei Chu[c], Nirwan Ansari[*,d]

[a] Department of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
[b] Wuhu Overseas Students Pioneer Park, Wuhu, 241006, China
[c] Department of Electrical Engineering, Shanghai Jiaotong University, Shanghai 200240, China
[d] Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07029, USA

ARTICLE INFO

Keywords:
Joint localization of multiple sources
EDM
LRMC
SDP
WSN
WLAN

ABSTRACT

One major challenge of massive wireless networks is to identify the locations of source nodes from partially observed and noisy distance information. This is especially important for wireless sensor networks (WSN) and wireless local area networks (WLAN). In this paper, we propose a unified localization framework of multiple sources from an Euclidean distance matrix (EDM) with noise and outliers both in WSN and WLAN scenarios. We first develop a semidefinite programming (SDP) based low rank matrix completion (LRMC) estimator by using the semidefinite embedding lemma to recover EDM. Based on our recovered EDM, two robust localization estimators, namely, semidefinite relaxation localization (SDRL) and weighted semidefinite relaxation localization (WSDRL), are derived to efficiently relax our non-convex localization problem into a convex one, and yield more accuarate location estimates. As compared with existing techniques, our proposed techniques are more robust to noise and outliers with higher accuracies both in EDM recovery and source localization. Simulations and real data experiments are included to evaluate the performance of the proposed algorithms by comparing them with some existing methods.

1. Introduction

Location awareness for various potential applications in wireless sensor networks (WSNs) and wireless local area networks (WLANs) is crucial, e.g., emergency preparedness, target tracking, monitoring, signal classification, energy-efficient routing, and smart home [1,2]. Localization accuracy is one of the main challenges in WSN and WLAN localization problems.

In a nutshell, a WSN system [3–5] always comprises a great deal of sensor nodes, which can be categorized as reference nodes and source nodes [6,7]. Generally speaking, the locations of reference nodes are known *a priori*, whereas the locations of source nodes are to be estimated. In a WLAN scenario, the reference nodes are called access points (APs), while the source nodes are user equipments (UEs) [8]. In order to discuss the range-based localization framework both in WSN and WLAN scenarios, here and subsequently, we just consider reference nodes and source nodes for uniformity.

Most multiple source localization techniques, for instance, multi-dimensional scalar (MDS) [9–11] and its deviations [12], assume that a complete and exact Euclidean distance matrix (EDM) is needed. Obviously, this assumption does not hold in practice due to the existence of noise and outliers, which may be attributed to limited communication ranges, Non-Line-Of-Sight (NLOS) propagation [13], power failure, and strong interference [14]. All these factors may result in missing data or large "errors" in EDM, known as *outliers*. It has been demonstrated that even a small number of outliers can degrade the localization accuracy in WSN and WLAN drastically [15,16].

To tackle the influence of noise and outliers, Oğuz-Ekim et al. [17] considered the simultaneous localization and tracking (SLAT) of single target problem in the WSN environment by using EDM in presence of noise and outliers. The proposed approach needs to estimate the outline of network configuration and prior knowledge about the source position. Localization of multiple sources using noisy EDM has also been discusssed in [1,18–23], but these works did not consider the influence of outliers. Additionally, most of these works just focused in the localization problem in the WSN environment.

In this paper, we consider the problem of jointly localizing multiple sources from an incomplete and noisy EDM in both WSN and WLAN

☆ This work has been supported in part by the National Natural Science Foundation of China under Grant (Nos. 61371184, 61671137, 61771114, 61771316) and the Fundamental Research Funds for the Central Universities (No. ZYGX2016J028).
* Corresponding author.
E-mail addresses: xsguo@uestc.edu.cn (X. Guo), LeoChu@sjtu.edu.cn (L. Chu), nirwan.ansari@njit.edu (N. Ansari).

https://doi.org/10.1016/j.comcom.2018.03.007
Received 29 September 2017; Received in revised form 16 November 2017; Accepted 11 March 2018
Available online 15 March 2018

scenarios. Considering that EDM is a low rank but not a PSD matrix with zero diagonal, we develop a semidefinite programming (SDP) based low rank matrix completion (LRMC) estimator by employing the semidefinite embedding lemma. The proposed SDP based LRMC approach is more robust to outliers and noise with lower relative completion errors (RCEs). Based on the recoverd EDM, two more robust and accurate estimators: a semidefinite relaxation localization (SDRL) estimator and a weighted semidefinite relaxation localization (WSDRL) estimator, are proposed to finalize the location estimates of multiple sources. The efficacy of our proposed algorithms are verified through simulation and real data results both in WSN and WLAN scenarios. The main contributions of this work is to derive a unified localization framework of multiple sources from a Euclidean distance matrix (EDM) with noise and outliers both in WSN and WLAN scenarios, as summarized below:

(1) First, we define a new outlier rate, apply it for both WSN and WLAN scenarios, and analyze theoretically the differences of the outlier rates of the two scenarios. Simulations and real data results are also conducted and demonstrated to conform with the theoretical analysis.
(2) For an incomplete and noisy EDM, we propose a robust SDP based matrix recovery approach by using semidefinite embedding lemma. This method can avoid iterative processing with higher accuracy in RCEs. Our method does not need to estimate the initial positions of sources and the outline of network configuration. We show its competitive performance in WSN and WLAN scenarios with higher outlier rates and noise levels by several simulation and real data experimental comparisons.
(3) Based on the recovered EDM, we derive two robust localization algorithms, SDRL and WSDRL, by transforming a nonconvex problem into a convex one. The remarkable performance improvement of our two estimators is attributed to their making full use of the distance measurements in EDM as compared with CMDS and DWMDS. The robustness of SDRL and WSDRL can decrease the accuracy demand of outlier detection in EDM recovery.

2. Related works

EDM is composed of distance information among all nodes, including source nodes and reference nodes, deployed in the two typical wireless networks. Basically, EDM can be inferred from the commonly used measurements (RSS [24], TOA [13,25], TDOA [26], AOA [27], and some combinations of them [28]). It is very challenging to obtain an accurate and robust location estimate from the incomplete and noisy EDM. Fortunately, since the noiseless EDM is a low rank matrix, low rank matrix completion framework can be utilized to tackle the problem. Some existing state-of-the-art low rank matrix completion approaches, including the exact completion solution (ECS) approach [29], singular value thresholding (SVT) [30], augmented Lagrange multiplier (ALM) [31], and accelerated proximal gradient (APG) [32], can be used to complete the EDM. In the noiseless case, ECS is a unique and exact solution when complete ranges between source and reference nodes are specified, but it performs poorly when EDM is corrupted. Alternatively, with EDM being a low rank matrix, the EDM completion problem (EDMCP) can be handled equivalently by low rank matrix completion (LRMC) solutions. The SVT method is also a simple LRMC solution, but it needs to compute the singular value decomposition (SVD) of the object matrix and drops the singular values, which are below a threshold, at each iteration, and this may lead to slow convergence and poor completion performance. The major drawbacks of these existing methods include: (1) they are highly sensitive to the choice of the initial point, (2) they converge very slowly, and (3) they do not provide any information on the global minimum [33].

Given the completed range information, the classical multi-dimensional scaling (CMDS) [9] and distributed weighted multi-dimensional scaling (DWMDS) [22] are commonly used to estimate the positions of sources. The goal of CMDS is to find a low dimensional representation of a group of objects such that the distances between objects fit as close as possible to a given set of distances. CMDS needs a complete and accurate distance matrix to estimate source locations, and is thus impractical in real scenarios. Besides, CMDS is a closed-form solution with poor noise performance and requires the centroid of all the nodes to be at the origin. DWMDS adds a penalty term to the cost function to account for prior knowledge about node locations, and so it works better in the presence of noise. However, in some sense, it just makes full use of all the existing measurements because its cost function does not take the missing measurements into account. Comparatively speaking, semidefinite relaxation is a good strategy when the completed range information is given; a good example of semidefinite relaxation localization method based on two stages signal strength difference has been addressed in our previous work [1].

3. Problem formulation

3.1. Measurements model

Consider a set of M reference nodes and N source nodes in a 2-dimensional location space. Let $\boldsymbol{u}_j = [a_j, b_j]^{\mathrm{T}}, \ j = 1, 2, \dots, M$ and $\boldsymbol{x}_i = [x_i, y_i]^{\mathrm{T}}, i = 1, 2, \dots, N$ be the coordinates of the jth reference node and the ith source node, respectively. The location vectors of reference nodes and source nodes can be denoted as $\boldsymbol{u} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_M]$ and $\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N]$, respectively. Also note that 3-dimensional localization can be handled in a similar manner. The measurements are used to infer range information among nodes for localization. The noiseless distance between the ith reference node and jth source node, denoted by d_{ij}, is

$$d_{ij} = \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \tag{1}$$

Let ensembles $\tilde{d}_{ij}$ be the noisy distances among all nodes. The problem of localizing multiple sources is then to find a realization of $\boldsymbol{x}$ such that

$$\begin{aligned} \|\boldsymbol{u}_i - \boldsymbol{x}_j\| &= \tilde{d}_{ij} = d_{ij} + n_{ij}, (i, j) \in \mathscr{A}, \\ \|\boldsymbol{x}_j - \boldsymbol{x}_k\| &= \tilde{d}_{jk} = d_{jk} + n_{jk}, (j, k) \in \mathscr{B}, \end{aligned} \tag{2}$$

where $\mathscr{A}$ and $\mathscr{B}$ are the indices of reference/source and source/source, and n_{ij} and n_{jk} denote range measurement errors which are commonly modeled as independent zero-mean Gaussian variables with variances σ_{ij}^2 and σ_{jk}^2, respectively [22,34].

Research studies show that the localization estimator based on (2) is nonconvex whose performance highly depends on the required initial settings [9,22]. An alternative methodology is to adopt a squared range measurements instead of the range measurements (e.g., [34,35] and the references therein). Specifically, the localization problem in (2) by using the squared range measurements can be approximately expressed as

$$\begin{aligned} \|\boldsymbol{u}_i - \boldsymbol{x}_j\|^2 &= d_{ij}^2 + w_{ij}, (i, j) \in \mathscr{A}, \\ \|\boldsymbol{x}_j - \boldsymbol{x}_k\|^2 &= d_{ik}^2 + w_{jk}, (j, k) \in \mathscr{B}, \end{aligned} \tag{3}$$

where w_{ij} and w_{jk} are the squared range measurement errors, which are *approximately* regarded as independent zero mean Gaussian variables with variances $\sigma_{w_{ij}}^2$ and $\sigma_{w_{jk}}^2$ [35]. It has been proven that the squared range methodology problem is also nonconvex but can be globally solved by semidefinite relaxation (SDR) [35]. For the convenience of the algorithm development, we give an EDM representation of the squared range in (3) as follows.

3.2. EDM representation and properties

EDM, which represents range information among all nodes, is a