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Analytical modeling and analysis of interleaving on correlated wireless channels

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ABSTRACT

Interleaving is a mechanism universally used in wireless access technologies to alleviate the effect of channel correlation. In spite of its wide adoption, to the best of our knowledge, there are no analytical models proposed so far. In this paper we fill this void proposing three different models of interleaving. Two of these models are based on numerical algorithms while one of them allows for closed-form expression for packet error probability. Although we use block codes with hard decoding to specify the models our modeling principles are applicable to all forward error correction codes as long as there exists a functional relationship (possibly, probabilistic) between the number of incorrectly received bits in a codeword and the codeword error probability. We evaluate accuracy of our models showing that the worst case prediction is limited by 50% across a wide range of input parameters. Finally, we study the effect of interleaving in detail demonstrating how it varies with channel correlation, bit error rate and error correction capability. Numerical results reported in this paper allows to identify the optimal value of the interleaving depth that need to be used for a channel with a given degree of correlation. The reference implementations of the models are available [1].

1. Introduction

In spite of significant progress in hardware design and associated signal processing algorithms made over the last decades wireless channels still remain prone to transmission errors. The reason is that reliability and hardware complexity is often traded for additional capacity at the air interface. Forward error correction (FEC) codes along with retransmission techniques are supposed to be responsible for concealing residual errors. FEC codes are nowadays used in most modern wireless access technologies.

It is well documented that FEC codes show their best performance when bit errors happen at random without any sort of dependence between them. On the other hand, wireless channels are known to exhibit a high degree of correlation manifesting itself in clipping of bit errors. Although there are codes that may tolerate a certain degree of error clipping (e.g. Reed-Solomon codes) correlated channel statistics lead to their sub-optimal performance.

In spite of universal usage of interleaving, to the best of our knowledge there are no analytical models capturing its performance. The reason is twofold. First, most studies of wireless channel performance have been carried out assuming that the correlation in the bit error process is effectively removed using interleaving. In those investigations, where the correlation in the bit error process has been explicitly assumed, no interleaving functionality was considered. System level simulators that are widely used nowadays to evaluate performance of wireless access technologies may include interleaving as a basic block at the physical layer. However, interleaving is still implemented in rather unguided manner, i.e. the interleaving depth is set to some default value. Due to these reasons, interleaving is regarded as one of the features of wireless channels prohibiting their detailed cross-layer analytical studies. In this paper we will fill this void proposing three analytical models for interleaving having different degree of complexity and accuracy. Our models are suitable for any FEC codes with hard decoding that can correct k bits (symbols) in a codeword of length n. This is the case for block codes such as BCH and RS codes. Minor modifications are required for codes with soft decoding, e.g. RS or turbo codes. Nevertheless, the models are applicable as long as there is relationship, possibly probabilistic, between the number of incorrectly received bits in a codeword and the codeword error probability.

The rest of the paper is organized as follows. In Section 2 we highlight the importance of accurate modeling of interleaving process

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Table 1
Notation used in the paper.

Parameter	Definition
Δt	Time to transmit a single bit
I	Number of codewords
n	Length of the codewords in bits
k	Number of data bits in a codewords
l	Number of data bits that can be corrected
M	Number of interleaved codeblocks in a packet
$c_j^{(i)}$	Bit j of codeword i
$\{S(u)\}$	Bit error process
α, β	Transition probabilities of bit error process
c	Lag-1 NACF of the bit error process
p_E	Bit error probability
p_I	Interleaved codeblock error probability
p	Packet error probability
p_C	Codeword error probability
c_C	Lag-1 NACF of the codeword error process
ν_{00}, ν_{11}	Probability of correct/incorrect first two codewords
$f_1(1)$, $f_2(1)$	Bit error probability is states 1 and 2
D	Transition matrix of the bit error process
$D(1)$, $D(0)$	Transition matrices with and without error
$A(0)$, $A(1)$	Supplementary matrices, $A(0) + A(1) = 1_I$
1_I	Identity matrix
$D(i, n)$	nI-steps matrix with i incorrect bits
$D(i, j, n)$	nI-steps matrix with i and j incorrect bits
$\vec{h}$	Initial state probability vector
$\vec{v}$	Transitions rates from transient to absorbing state
P	Rate matrix of the absorbing chain
Q	Rate matrix of transient states
q_{ij}	Transition rate between state i and j
$\vec{0}$	Zero vector of appropriate size
T	Time till absorption
$F_T(k)$	CDF of time till absorption in k steps
$\vec{\pi}_C$	Stationary distribution of the codeword process
$D_C(0)$	Transition matrix of codeword process with error
α_C, β_C	Transition probabilities of codeword error process
$\Phi(x)$	Error function
μ, σ	Mean and variance of p_I estimate
$\hat{p}$	Estimate of the packet error probability
N	Number of experiments
γ	Confidence probability
I_L	Indicator of the packet loss event

and introduce the system model we work with. Further, in Section 3, three models for interleaving having different degree of complexity and accuracy are introduced. We assess accuracy of the models in Section 4. The effect of interleaving is analyzed in detail in Section 5. Conclusions are given in the last section.

2. System model

The notation used in the paper is provided in Table 1.

2.1. Wireless channel model

In this study, to make the model universally applicable we abstract the channel organization mechanisms assuming the bit error statistics as the input to the model.

We consider the bit error process as a covariance stationary binary process, where 1 and 0 denote incorrect and correct bit reception, respectively. Consider a discrete-time environment with constant slot duration Δt corresponding to the amount of time required to transmit a single bit over a wireless channel. We model the bit error process using the discrete-time Markov modulated process with irreducible aperiodic Markov chain $\{S(u), u = 0, 1, \cdots\}$, $S(u) \in \{0, 1\}$. When at most single event is allowed to occur in a slot this process is known as switched Bernoulli process (SBP). To parameterize a covariance stationary binary process only mean value and lag-1 normalized autocorrelation (NACF) coefficient have to be captured. It was shown in [2] that there is a special case of SBP called interrupted Bernoulli process (IBP) exactly matching mean and lag-1 NACF value. It is given by

$$\begin{cases} \alpha = (1-c)p_E \\ \beta = (1-c)(1-p_E) \end{cases} \quad \begin{cases} f_1(1) = 0 \\ f_2(1) = 1 \end{cases}, \tag{1}$$

where $f_1(1)$ and $f_2(1)$ are bit error probabilities in states 1 and 2, respectively, α and β are transition probabilities from state 1 to state 2 and from state 2 to state 1, respectively, c is the lag-1 NACF value of bit error observations, p_E is the BER. Letting D be transition probability matrix and defining the following matrices

$$A(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A(1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

the model is described by two matrices $D(0) = D \times A(0)$ and $D(1) = D \times A(1)$ describing transitions between states of the model with correct and incorrect reception of a bit. Since $A(0) + A(1) = 1_I$, where 1_I is the identity matrix.

Note that the channel model introduced above may not exactly capture correlation properties of various propagation environments. However, in practical situations, exact behavior of NACF is not known. The proposed model provide the so-called "first-order" approximation of correlated channel behavior. A special advantage of the model is that it can be easily tuned to explore the qualitative and quantitative effects of correlation. Still the extension of the model to the case of general finite-state Markov chain (FSMC, [3]) is straightforward. Indeed, irrespective of the number of states used to represent the bit error process and probabilities of bit errors in each state we can always partition D into $D(0)$ and $D(1)$ such that $D = D(0) + D(1)$. Finally, if wireless channels conditions exhibit piecewise stationary behavior as was reported in a number of studies (see e.g. [4,5]), this model may represent statistical characteristics of covariance stationary parts. In this case, (1) is interpreted as a model for a limited duration of time during which mean value and NACF of bit error observations remain constant.

2.2. Interleaving process

Assume that data are encoded into I, $I > 0$, codewords of the same length n. We denote these codewords by $c^{(i)}$, $i = 1, 2, \cdots$ where bit j of ith codeword is $c_j^{(i)}$. Without interleaving these bits would have been sent as

$$c_1^{(1)}, c_2^{(1)}, \cdots, c_n^{(1)}, \cdots, c_1^{(I)}, \cdots, c_n^{(I)}, \tag{3}$$

and in case of a deep channel fade we may incorrectly receive a significant amount of bits belonging to the same codeword. This may eventually lead to inability to decode this codeword even when the overall "average" channel quality is acceptable.

Let us now introduce interleaving of depth I, $I > 1$. According to the concept, we first combine I codewords into a matrix of size $n \times I$, where each codeword represents a row, i.e.

$$\begin{pmatrix} c_1^{(1)} & c_2^{(1)} & c_3^{(1)} & \cdots & c_n^{(1)} \\ c_1^{(2)} & c_2^{(2)} & c_3^{(2)} & \cdots & c_n^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_1^{(I)} & c_2^{(I)} & c_3^{(2)} & \cdots & c_n^{(I)} \end{pmatrix} \tag{4}$$

Given the matrix (4), we perform its column-wise transmission, i.e. column n is transmitted first with $c_1^{(1)}$ being the first bit sent, then column $n-1$ starting with $c_2^{(1)}$, etc. The transmitted sequence of bits looks as

$$c_1^{(1)}, c_1^{(2)}, \cdots, c_1^{(I)}, \cdots, c_n^{(1)}, c_n^{(2)}, \cdots, c_n^{(I)}. \tag{5}$$