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Wireless scheduling with multiple data rates: From physical interference to disk graphs☆

Olga Goussevskaia, Luiz F.M. Vieira*, Marcos A.M. Vieira

Computer Science Department, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil

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ABSTRACT

Scheduling of wireless transmissions is a core component of performance optimization of wireless ad-hoc networks. Current radio technologies offer multi-rate transmission capability, which allows to increase the network's throughput. Nevertheless, most approximability results of scheduling algorithms have focused on single-rate radios. In this paper, we propose two formulations for the problem of scheduling wireless requests with multiple data-rates, considering the physical interference model with uniform power assignment. The objective of both problems is to select a subset of communication requests to transmit simultaneously, such that the sum of their data rates is maximized and no collisions occur. In the first formulation, data-rates are given as part of the input. In the second formulation, the data-rate assignment is part of the solution. We show that, under certain constraints on the input, these problems can be approximated by a disk graph model. This means that, despite the global nature of the physical interference model, conflicts between simultaneous requests can be restricted to the local neighborhood of the transmitting nodes. We show how to build the corresponding disk graph instances and prove that a weighted maximum independent set in this graph-based model provides a constant-factor approximation in the physical interference model. Moreover, we implement a polynomial-time approximation scheme, as well as a parallel implementation of the algorithm, to obtain solutions that are within an arbitrarily small factor of being optimal in the disk graph model.

1. Introduction

Scheduling of wireless communication requests lies in the heart of performance optimization of wireless ad-hoc networks. Current wireless communication technologies, such as IEEE 802.11a/b/g/n and IEEE 802.16, allow data to be transmitted at multiple data rates. The higher the requested rate, the higher must be the signal-to-interference-plus-noise ratio at the receiver, which can be achieved either by increasing the transmitting power of the sender or by decreasing the interference of concurrent transmissions. Either way, the multi-rate functionality alters the spatial reuse constraints of the wireless channel, which in turn modifies the structure of the scheduling process of communication requests.

In this work we are interested in modelling and analyzing algorithms to solve the following problem. Given a set of wireless links that can transmit with multiple data rates, we want to select a subset of theses links, such that all of the selected receivers can successfully decode their messages and the overall data rate is provably close to the maximum possible, i.e., provide approximation guarantees for the obtained solutions. We propose two formulations for this problem. In the first formulation, referred to as the *Multi-Rate Scheduling Problem*, the data-rates are given as part of the input. In the second formulation, referred to as the *Variable-Rate Scheduling Problem*, the assignment of a data-rate to each link selected to transmit is part of the solution. Note that there is a trade-off between the total communication data-rate and the number of scheduled requests. Setting a communication request to a higher data-rate requires lower interference coefficient, which results in fewer number of concurrent transmissions. Setting a communication request to a lower data-rate results in fewer bits transmitted.

In this work we use two different interference models to analyze and solve these problems. On the one hand, we want to provide provably feasible solutions in a model as realistic as possible. On the other hand, we would like to be able to use a simple enough model to be able to derive concise theoretical bounds for our results. We start by defining the problems in the *physical interference model*. In this model, a transmission is considered successful iff the signal-to-interference-plus-noise ratio (SINR) at each

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* Corresponding author.
E-mail addresses: olga@dcc.ufmg.br (O. Goussevskaia), lfvieira@dcc.ufmg.br (L.F.M. Vieira), mmvieira@dcc.ufmg.br (M.A.M. Vieira).

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receiver is above a certain threshold, which depends on the radio hardware and the data-rate at which the transmission must be scheduled. This model is among the most realistic ones used in theoretical studies, but is also quite complex, making the process of designing algorithms quite challenging and does not allow the application of many existing analytical tools, such as, for example, graph-theoretic techniques.

Recently, it has been shown that some wireless link scheduling problems in the physical model can be approximated, to within constant factors, by graph-based models, provided that some input parameters are restricted. For example, in [3], it was shown that single-rate transmission conflicts in the physical model can be approximated by a *unit disk graph*, as long as constant transmission power is used and link lengths differ at most by a factor of 2. Such model transformation allows one to simplify the global and cumulative nature of wireless interference, as captured by the physical model, by using local, distance-based relations/edges of a geometric graph.

In this work, we build upon these results and use the "physical-to-graph model approximation" technique to solve wireless scheduling problems. By constructing a *disk graph* representation of each problem instance, we show that if a maximum weight independent set (MWIS) is computed on the disk graph, it provides a constant approximation to each problem in the physical interference model. We solve the problems by implementing a polynomial-time approximation scheme (PTAS) (as well as a parallel version of it). As opposed to many previous results in wireless networks that use graph-based models, this approach allows the application of graph-theoretic algorithmic tools, while guaranteeing feasible solutions and approximation bounds in the physical interference model, which is closer to reality.

The contributions of this work can be summarized as follows. Firstly, we present the model transformation process, showing how a graph-based problem instance can be built for the *Multi-Rate* and the *Variable-Rate Scheduling Problems*, keeping the desired SINR properties of the original model. Next, we prove that a solution to the MWIS problem on the graph instance provides a constant approximation to the original problem. This is an important step, since it allows to apply many existing graph-based algorithmic tools to the physical interference model, preserving approximation guarantees. Finally, we describe a PTAS solution for both problems, and show that good-quality solutions can be obtained.

This paper is structured in the following way. In Section 2 we describe some related work. In Section 3 we present the details of the modelling process and problem definitions. In Section 4 we show how to build a problem instance of the *Multi-Rate Scheduling Problem* in the disk graph model and prove that this model reduction is correct and guarantees a constant-factor approximation. In Section 5 we describe the model transformation process for the *Variable-Rate Scheduling Problem*. In Section 6 we describe the PTAS implementation. Finally, in Section 7 we present extensive simulation results.

2. Related work

Studying wireless networks in graph-based models usually involves studying the problems of coloring and independent set. Coloring a general graph is not only an NP-complete problem, but is also hard to approximate to within factor of $n^{1-\epsilon}$, for any constant $\epsilon > 0$ [4]. Wireless networks, however, can usually be better modeled by more restricted classes of graphs, such as geometric graphs. In [5], Clark et al. proved that a series of closely related problems to scheduling of wireless links, such as coloring in graphs, independent set, domination, independent domination, and connected domination, are NP-complete in unit disk graphs (UDGs). The maximum independent set problem can be approximated to within factor 5 using an online greedy algorithm [6] (which is optimum for an online deterministic algorithm) and factor 3 by a greedy offline algorithm [7]. In the case of disk graphs, the online greedy approach yields a $(n-1)$-approximation, whereas the greedy offline algorithm that processes disks in non-decreasing size order achieves a 5-approximation. Finally, the problem of maximum independent set in (unit) disk graphs can be approximated to within a factor $(1-\epsilon)$, for any fixed $\epsilon > 0$ using a polynomial-time approximation scheme (PTAS) [8,9]. In particular, Erlebach et al. [9] provide a PTAS for the weighted independent set problem in disk graphs. A short survey on disk graphs can be found in [10].

Analytical results in more realistic models, such as the physical interference model, are more recent. In [11] it was shown that the problem of scheduling wireless transmissions with uniform power assignment is NP-complete in the physical interference model. In [12], a constant approximation algorithm was proposed for the (maximization) one-slot scheduling problem and a logarithmic approximation for the (minimization) multi-slot problem. These results were derived for the single-rate scenario. Several other works studied different aspects of the problem, such as linear power assignment [13] and power control [14,15].

In [3] it was shown that single-rate scheduling in the SINR model can be approximated to within a constant factor by coloring a unit disk graph, in the case when constant transmission power is used and the link set is "nearly-equilength", i.e., link lengths vary by at most a factor of two. In this work we take a step further, and show that a constant approximation can be achieved by approximating multi-rate scheduling with a disk graph with variable disk radii.

Joint channel assignment and routing has been investigated in [16]. Efficient channel assignment schemes can greatly relieve the interference effect of simultaneous transmissions while routing schemes can alleviate potential congestion. In [17], Kodialam et al. characterized the capacity region in multi-radio multichannel wireless mesh networks and derived the upper bounds on the capacity in terms of achievable throughput. We solve the link scheduling problem, which is a fundamental problem in any wireless network. It is a building block that can be integrated into larger problems. For instance, it appears as a sub-problem in other problems, such as routing.

In [18], several versions of the wireless capacity problem were analyzed. Among them, one is referred to as "scheduling with QoS generalization and uniform power", and is similar to the *Multi-Rate Scheduling Problem* studied here. A constant approximation was obtained for the case of arbitrary link lengths and general metric space. In [19], the problems of power control and data-rate maximization are studied in one formulation. The proposed solution is proved to be $O(\log n)$-approximation and computes both the power levels and the data-rates assignment to the selected links. Our work, on the other hand, provides a constant approximation, although to a more restricted range of scenarios, and considers uniform power assignment, which one might argue represents most practical scenarios.

3. Model

In this work we study the problem of scheduling wireless communication requests (links) in the physical interference model using multiple and variable data rates.

A typical problem instance consists of a set $L = \{\ell_1, \dots, \ell_n\}$ of n wireless links (and, as explained later, a set of data rates), where each link ℓ_i represents a communication request from a sender s_i to a receiver r_i: $\ell_i = (s_i, r_i)$. The communication devices are viewed as nodes positioned in a Euclidean space. The distance between