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One-parameter fractional linear prediction☆

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ABSTRACT

The one-parameter fractional linear prediction (FLP) is presented and the closed-form expressions for the evaluation of FLP coefficients are derived. Contrary to the classical first-order linear prediction (LP) that uses one previous sample and one predictor coefficient, the one-parameter FLP model is derived using the memory of two, three or four samples, while not increasing the number of predictor coefficients. The first-order LP is only a special case of the proposed one-parameter FLP when the order of fractional derivative tends to zero. Based on the numerical experiments using test signals (sine test waves), and real-data signals (speech and electrocardiogram), the hypothesis for estimating the fractional derivative order used in the model is given. The one-parameter FLP outperforms the classical first-order LP in terms of the prediction gain, having comparable performance with the second-order LP, although using one predictor coefficient less.

1. Introduction

Prediction in signal processing is a mathematical model where future values of a discrete-time signal are estimated based on the previous signal values. In linear prediction it is equivalent to finding the output of a linear time-invariant (LTI) filter by observing only the previous output samples. A linear predictor is said to have an order of p if the filter has p taps. The task of linear prediction is to determine a set of filter coefficients which best describe the behavior of an LTI system.

Linear prediction is a fundamental tool used in many diverse areas, such as audio signal processing [1], image processing [2], object detection and object tracking [3], electrocardiogram (ECG) signal processing [4] and sensor networks [5]. Probably the most common application is modeling, compression, coding and generation of speech signal [6].

Signal prediction presumes using the prior history of the signal. One would assume that the longer the signal history is available for the predictor evaluation, the better prediction is obtained. However, there are applications, such as e.g. image coding, where involving longer signal history is not beneficial, as increasing the prediction order does not improve the performance. It is because the images are typically the first-order Markov processes, meaning that the current pixel is dependent only on the previous pixel (i.e. the first-order prediction is used [7]). Similar situation can be observed in video coding, where the performance improvement becomes insignificant after the second-order prediction [8]. Even in speech compression the first-order and the second-order prediction provide a substantial increase in the performance, while higher-order predictors provide relatively little improvement.

Fractional derivatives were successfully used for modeling different physical processes instead of commonly used models involving integer-order derivatives. Although the foundation of the fractional calculus goes back to 17th century, it has been only in the

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last decades that the topic has been rediscovered in mathematics [9], physics [10], control theory [11], viscoelasticity [12], signal processing [13–15], and many other areas.

Recently, fractional linear prediction (FLP) has attracted attention in different fields of signal processing, such as detection and discrimination of premature ventricular contractions in ECG signals in [16], electroencephalogram (EEG) signal modeling in [17], and speech signal modeling in [18]. However, in the three above-mentioned papers the authors do not explain the methodology for estimating the orders of fractional derivatives, but simply choose arbitrary values. Also, the transition from linear to fractional linear prediction, i.e. the motivation for introducing the fractional derivatives in their models is not completely clear.

In this paper we focus on investigation of one-parameter fractional linear prediction, showing that the classical first-order linear prediction is only a special case of the proposed model. We begin by expanding the signal into Taylor series, which is a sum of terms calculated from the values of the function's (integer-order) derivatives at a single point, and deriving classical linear prediction from this representation by truncating the Taylor series to only the first term (obtaining the first-order linear predictor) or to the first two terms (obtaining the second-order linear predictor).

This idea is further elaborated using the generalized (fractional) Taylor series [19–21], instead of the classical Taylor series expansion, leading to the model of one-parameter fractional linear prediction proposed in this paper. The closed-form expressions for the evaluation of the one-parameter FLP coefficients are derived. Contrary to the first-order linear predictor that uses only one previous sample, the proposed one-parameter FLP requires the memory of at least two signal samples, while the number of predictor coefficients that have to be optimized is still equal to one; hence the complexity of the predictor is not increased. Moreover, the analysis of the relation between the number of samples used to evaluate the proposed FLP model and the order of the fractional derivative is given in the paper.

The remainder of the paper is organized as follows. Section 2 gives preliminaries on the fractional derivative operator and briefly recalls the optimal predictor design of the first-order and the second-order linear predictor. In Section 3 the one-parameter FLP is presented, that is derived from the generalized Taylor series. In Section 4 the proposed one-parameter FLP model is analyzed using two different test signals (one of them being the synthesized musical chord), followed by the results of speech and ECG signal modeling. Finally, the concluding remarks are given in Section 5.

2. Preliminaries

2.1. The fractional derivative operator

There are many different definitions of the fractional-order differentiator. In this paper the Grünwald–Letnikov (GL) definition that is widely used for the numerical solution of fractional derivative of a function $x(t)$ at time instant t is adopted [22]:

$$ {}_aD_t^{\alpha}x(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{K=\left[\frac{t-a}{h}\right]}(-1)^j\binom{\alpha}{j}x(t-jh), \tag{1} $$

where K is the integer part of the fraction $(t-a)/h$, a and t are lower and upper terminals of differentiation, respectively, and $\alpha\in\mathbb{R}$ is the arbitrary real order of the fractional derivative. Note that in fractional derivatives limits must be considered, they vanish only in integer-order derivatives. In case the step-size h is very small, or $t\gg a$, the number of terms in (1) becomes very large, what might lead to problems with computational efficiency in real-time implementations. Based on the fact that for large t the history of the function close to the lower terminal $t=a$ does not influence the behavior of the function considerably, only recent past of the function can be taken into account (the "short-memory" principle) [22].

The binomial coefficients in (1):

$$ \omega_j^{(\alpha)} = (-1)^j\binom{\alpha}{j}, \quad j=0,1,2,\ldots, \tag{2} $$

can be estimated using the recurrent relationship:

$$ \omega_0^{(\alpha)} = 1;\quad \omega_j^{(\alpha)} = \left(1-\frac{\alpha+1}{j}\right)\omega_{j-1}^{(\alpha)},\quad j=1,2,\ldots. \tag{3} $$

For more detailed review of the fractional-order operator definitions the reader is referred to [22].

2.2. Optimal linear predictor design

Considering a continuous signal $x(t)$, such that all its higher-order derivatives exist at any arbitrary time t, that is sampled at small time interval between the successive samples h (sampling period), we can expand the signal at the next sampling instance $x(t+h)$ using the Taylor series:

$$ x(t+h) = \sum_{k=0}^{\infty}\frac{h^k}{k!}x^{(k)}(t) = x(t) + \frac{h}{1!}x'(t) + \frac{h^2}{2!}x''(t) + \frac{h^3}{3!}x'''(t) + \ldots. \tag{4} $$

Truncating the Taylor series (4) to the first two terms one can roughly predict a future signal at time $t+h$, knowing only the