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A new signal reconstruction method in compressed sensing☆

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ABSTRACT

Aiming at the shortcomings of being rough by using the l_1 norm to replace the original signal for signal reconstruction, this paper first constructs the l_1 norm's smooth approximation function and studies the monotonicity and convergence of the optimal solution sequence. Second, the pseudo-function $||x||_p$ $(0<p<1)$ replaces $||x||_1$ in signal reconstruction, and the maximum entropy function is used to smooth l_p $(0<p<1)$. In this paper, the algorithm's performance is analyzed and its feasibility is verified using one- and two-dimensional signal reconstruction. This algorithm has smaller error than other compression reconstruction algorithms, better structure, and a more convenient calculation.

1. Introduction

Previously, signal processing based on the traditional Nyquist sampling theorem has consisted of three parts: sampling, transform, and compression. A signal-reconstruction algorithm is used to restore the original information signal after transmission. Determining techniques for improved signal sampling and recovery has become the main direction of study. In recent years, compressed sensing [1–4] has been explored as a new signal sampling and recovery theory. With its obvious advantages in modern signal processing, it is widely used in applied mathematics, computer science, electronic engineering, aerospace engineering, medical imaging, radar detection, digital image processing, and many other areas. Scholars have researched it extensively [4]. It has been proved that the signal can be reconstructed accurately by solving the linear optimization minimum norm l_0 problem because it has the sparsest solution with the optimal sparse approximation. However, this is an NP-hard problem, i.e., its solution entails the examination of all possible permutations and combinations of nonzero values in the original signal. The computational complexity is huge, and the problem cannot be solved directly. It has been proved that under certain conditions the minimum norm l_1 problem and the minimum norm l_0 problem have the same solution [5]. Methods have been proposed for obtaining the solutions to the l_0 norm [6–11] and l_1 norm [12–14], with good results.

The outline of the paper is as follows First, the basic idea of norm l_1 is analyzed in Section 2, and the smooth approximation function is constructed. The monotonicity and the convergence of the optimal solution sequence are optimized. The effect is illustrated by a simple experiment. In Section 3, an algorithm based on the maximum entropy smoothing approximation l_p is proposed according to the previous section and the convergence is proved. In Section 4, the previous section is compared with the other algorithms from the experimental process, and one-dimensional signals and two-dimensional images are also compared from the reconstruction of metrics. Finally, Section 5 is the summary of the paper.

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2. Signal reconstruction based on smooth graduation l_1 norm

The ideal signal reconstruction is obtained by solving the original reconstruction model or the model based on the minimum l_0 norm However, this is an NP question, so it is converted to a solution to an l_1 minimum norm Since this norm is not smooth, this paper constructs a smooth graduation algorithm based on the l_1 norm, describes the monotonicity of the function and the convergence of the optimal solution, and uses this function to perform the signal reconstruction.

2.1. Basic knowledge

When solving the compressed sensing signal reconstruction, the l_0 norm solution is given as follows:

$$\begin{cases} \min & d(x) = ||x||_0 \\ s.t. & Ax = y \end{cases} \tag{1}$$

The l_0 norm is an NP question. It has been proved that signal reconstruction based on the minimum l_0 norm is equivalent to that based on solving the minimum l_1 norm [15,16]. Therefore, signal-reconstruction questions are handled by solving the minimum l_1 norm with the following model:

$$\begin{cases} \min & d(x) = ||x||_1 \\ s.t. & Ax = y \end{cases} \tag{2}$$

2.2. The improved l_1 question model

An algorithm based on the l_1 solution cannot be derived, so Eq. (2) cannot be solved by an algorithm based on massive derivation. Eq. (2) is a convex programming question that can be converted to one of linear programming. However, the size of the original question is doubled and the computing space is increased. A solution involving large-scale data is characterized by slow computation speed and poor signal-reconstruction effects. This paper adopts smooth, gradual, and progressive ideas, constructs a smoothing function based on the l_1 norm, studies the monotonicity and optimal sequence, and finally solves Eq. (2).

Assuming Definition 1, when $x \in R^N, t > 0$, then

$$F(x) = ||x||_1 = \sum_{i=1}^{N} |x_i|, F_t(x) = \sum_{i=1}^{N} \sqrt{x_i^2 + \frac{c}{t^2}} \tag{3}$$

Theorem 1. $\lim_{t\to\infty} F_t(x) = F(x), F_t(x) = \sum_{i=1}^{N} \sqrt{x_i^2 + \frac{c}{t^2}}, x \in R^N$

Proof.

$$\begin{aligned} F_t'(x) &= \sum_{i=1}^{N} \frac{1}{2\sqrt{x_i^2 + \frac{c}{t^2}}} \bullet \left(x_i^2 + \frac{c}{t^2}\right)' \\ &= \sum_{i=1}^{N} \frac{1}{2\sqrt{x_i^2 + \frac{c}{t^2}}} \bullet \left(\frac{-2c}{t^3}\right) \\ &= \sum_{i=1}^{N} \frac{-c}{t^3\sqrt{x_i^2 + \frac{c}{t^2}}} \\ &= \sum_{i=1}^{N} \frac{-c}{t^2\sqrt{(tx_i)^2 + c}} < 0 \end{aligned}$$

Then, $\{t_k\}$ is a monotonically decreasing integer sequence.

The following proves that $F_t(x)$ is bounded.

For any x and t

$$\begin{aligned} F_t(x) - F(x) &= \sum_{i=1}^{N} \sqrt{x_i^2 + \frac{c}{t^2}} - \sum_{i=1}^{N} |x_i| = \sum_{i=1}^{N} \left(\sqrt{x_i^2 + \frac{c}{t^2}} - \sqrt{x_i^2}\right) \\ &= \sum_{i=1}^{N} \frac{\frac{c}{t^2}}{\sqrt{x_i^2 + \frac{c}{t^2}} + \sqrt{x_i^2}} \le \sum_{i=1}^{N} \frac{\frac{c}{t^2}}{\frac{\sqrt{c}}{t}} \\ &= \frac{\sqrt{c}}{t} N \end{aligned}$$