Computers and Electrical Engineering 67 (2018) 114–133

Contents lists available at ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

Non-local total variation regularization models for image restoration☆

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ARTICLE INFO

Article history:
Received 31 October 2017
Revised 9 March 2018
Accepted 9 March 2018

MSC:
68U10
94A08

Keywords:
Non-local total variation
Total bounded variation
Rayleigh noise
Split Bergman
Variance stabilization
Linear blur

ABSTRACT

Restoration of images corrupted by data-correlated Rayleigh noise distribution has not been studied much extensively in the literature, unlike the other noise distributions. In this paper, we analyze the degradations due to a data-correlated Rayleigh noise and a linear blurring artifact. This work employs a variance stabilization approach and two variational approaches for restoring images from their noisy and blurred observations. The split-Bregman iterative scheme is used for numerically solving the models to improve their convergence rates. Furthermore, non-local total variation and non-local total bounded variation priors are being used as regularizers in these models to improve their restoration efficiency. Various synthetic and real images (such as ultrasound and synthetic aperture radar images) are tested to show the performance of these models.

1. Introduction

Image restoration under a data-independent noise has been studied in detail in the literature, see [1,2] and the references therein. However, such an elaborate analysis is not done much on the data-correlated or data-dependent noise. Nevertheless, much of the practical imaging modalities produce data-correlated noise. This has been a matter of concern over the last couple of decades. There are some models proposed for restoring images corrupted by data-correlated speckle noise [3–5] and Poisson noise (considering the intensity values generated as the outcome of a Poisson process), see [6] and the reference therein. Though there are some studies reported for Rayleigh noise removal in the literature (see [7–9]), not much extensive analysis relating to the distribution has been done. Besides, none of the models proposed so far for the Rayleigh noise distribution (which use the fast solver such as split-Bregman scheme) consider the linear blurring artefacts due to device related defects. Furthermore, as observed in some previous works, ultrasound and Synthetic Aperture Radar (SAR) images are generally corrupted by speckles (granule-like structures). Moreover, at a high scatter density these speckles are found to follow a Rayleigh distribution [10]. This motivates us to propose an efficient model to overcome the Rayleigh distributed speckles commonly found in ultrasound and SAR imagery.

☆ Reviews processed and recommended for publication to the Editor-in-Chief by Guest Editor Dr. Y. Zhang.

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https://doi.org/10.1016/j.compeleceng.2018.03.014

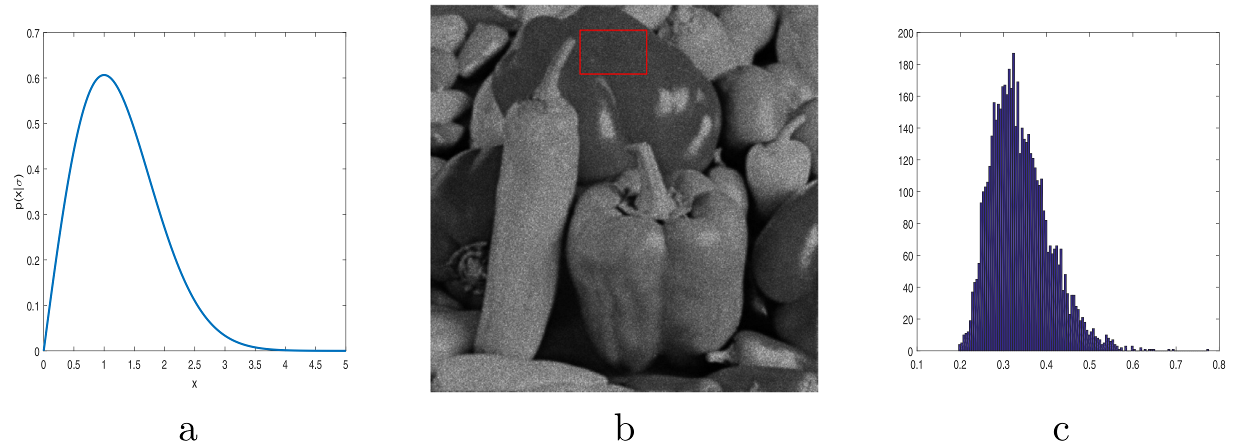

Fig. 1. (a) Rayleigh PDF with $\sigma = 1$, (b) Peppers image corrupted with Rayleigh noise of $variance = 0.1$ (c) Histogram of the homogeneous region highlighted in (b).

The Probability Density Function(PDF) of the Rayleigh distribution is given by,

$$p(x|\sigma) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}}, \tag{1}$$

where x is the random sample and σ is the scale parameter. In Fig. 1(a) the PDF of a Rayleigh distribution is shown. Fig. 1(b) shows the image “peppers” corrupted by a Rayleigh noise of $variance = 0.1$ and mean 1. The histogram of the noise extracted from a homogeneous intensity region in the image (shown using a red coloured rectangle) is plotted in Fig. 1(c).

Common methodologies adopted for image restoration in case of data-correlated noise are variance stabilizing transforms and MAP estimation models for the noise distribution. Variance stabilization methods, transform data to a domain where noise becomes un-correlated and MAP estimation models maximize the posterior probability of data. In this work, we analyze both these methods for Rayleigh noise in detail. Variance stabilizing transforms (VST) are proposed for Poisson and Poisson–Gaussian noise distributions in [11,12], respectively. Furthermore, a VST combined with non-local averaging is proposed for Rayleigh noise in [13]. MAP estimation methods are proposed for Gamma, Poisson and Rican distributions in [3,6,14], respectively. In these works (MAP based estimators) an energy functional is being derived based on the posterior probability of the density function. The Total Variation (TV) prior is being assumed in these works.

The rest of the paper is organized as follows. The major contributions of this paper are highlighted in Section 2. In Section 3 we discuss the variance stabilization transform for the Rayleigh distributed noise. In Section 4, different priors for variational frameworks existing in the literature are detailed with their definitions. Section 5 shows the formulation of the proposed variational models. Fast numerical schemes using the split-Bregman formulation for the proposed variational models are highlighted in Section 6 along with the algorithms. Experimental results and their analysis are provided in Section 7. Finally, we conclude the work in the last section.

2. Contributions of the paper

In this paper, we introduce three models for restoring images corrupted by data-correlated Rayleigh noise and linear blurring artefacts. The first one is based on a variance stabilization transform and a computationally efficient non-local TV regularization. The second and third ones are variational models with different regularization priors and their energy functional is designed based on the MAP estimate of the noise distribution. The first one among these variational models uses a non-local total variational regularization prior, whereas the second one uses a non-local total bounded variational regularization prior. Fast numerical solver using the well-known split-Bregman iterative scheme is employed for the implementation. The condition for the existence of a unique minimizer is analyzed.

3. Variance stabilizing transform

Stabilizing the variance makes noise un-correlated with the data. Subsequently, the distribution approaches an additive Gaussian [13]. The main idea behind stabilizing the variance is, defining a transform function that makes the variance a constant in the transformed domain. This function is assumed to be monotonic. The derivation of the variance stabilization transform (VST) for a Rayleigh distribution is provided in the A.1 of this paper.

Model I

The forward VST is (see A.1 for the details):

$$h(u) = \frac{1}{\sqrt{C}} \log(u), \tag{2}$$