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Incorporating fractional calculus in echo-cardiographic image denoising☆

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ABSTRACT

Speckle noise is an inherent property of echo-cardiographic images. It badly affects the diagnostic value of the image. This research is focused on mitigating speckle noise from echo-cardiographic images. Proposed denoising filter is based on fractional calculus, which is an emerging research topic in the field of image processing. The proposed methodology first divides pixels of noisy image into various regions i.e. homogeneous, texture and edge using a region classifier. This region classifier works on the basis of large eigenvalue of hessian matrix. Fractional calculus based filter mask is applied on each pixel of noisy image. Order of this mask is selected adaptively according to the membership of current pixel with image region. Simulations are carried out on real echo-cardiographic and standard images. Results of the proposed technique outperforms other recently proposed methods when compared on qualitative and quantitative basis using metrics, peak signal to noise ratio (PSNR), correlation coefficient (r), mean square error (MSE), edge preservation index and structural similarity (SSIM).

1. Introduction

Speckle noise, a multiplicative noise, corrupts ultrasound images during acquisition. As a result, physicians cannot analyze an image correctly and speedily. Therefore, denoising is an important preprocessing task. Many despeckling techniques have been proposed in literature. Lee [1], Kuan et al. [2] and Frost et al. [3] utilizes local statistics to denoise an image. These filters perform well in smooth areas but fail in edge region. Anisotropic diffusion [4,5] is diffusion method which is based on heat equation. It fails to preserves important details of an image. A denoising algorithm is presented which employed wavelets and nonlocal dictionary learning [6]. Nonlocal hierarchical sparse dictionary on the wavelet coefficients is prepared for noisy input image. Dictionary preparation makes this algorithm computationally complex. Bilateral filter replaces every pixel with the mean of nearby similar pixels [7]. Many images have similar patches scattered throughout the image. These similar patches are filtered together using on-local means filter [8]. High variance of noise brings down the performance of a filter. Fuzzy logic is good to handle uncertainty of speckle noise [9]. Babu et al. proposed an adaptive fuzzy logic based methodology that selects appropriate filer for every area of an image.

Introduction of fractional calculus [10–14] has revolutionized the research in the area of image processing. Fractional calculus involves fractional differentials and fractional integrals. Fractional order differentiation preserves the low frequency components like texture, while enhancing the high frequency components such as edges of image whereas fractional integration attenuates high

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frequency components like noise. Fractional differentials are suitable for image enhancement techniques and fractional integrals [15,16] are used in image denoising algorithms. Different orders of fractional differentials/integrals provide different amount of gain/attenuation. Therefore applying same order of fractional integration/differentiation on entire image is the main drawback of these techniques is that they use same fractional order integral for the whole image. Hu et al. [15] and Huang at el. [16] denoising techniques fail to achieve satisfactory denoising effect because they ignore the suitability of different fractional order curve in relation to image region. He et al. [17] proposed an improved fractional integration mask for denoising of digital images. This mask when convolved with the input image provides good denoising results. Its drawback is manually adjusted fixed order of fractional integration. Jalab et al. proposed a denoising technique using fractional Alexander polynomials (FAP) for image denoising [18]. Reisz fractional differential-based approach for texture enhancement in image processing has been proposed in [19] and [20]. These techniques used second-order Reisz fractional differential operator. Reisz fractional differential is calculated using integral function. Recently proposed fractional integral based technique [21] deploy local average gradient to select appropriate fractional order for each pixel of the image. However, this technique works for noise of small probability only. We present here a noise removal method based on fractional calculus and rough set theory. Image pixels are classified into different image areas and the boundary for each region is defined with the help of rough set theory [24]. Pixels are denoised by the fractional integration filter which adopts optimal orders of fractional integrals that results in good denoising.

Sequence of this paper is as follows: Section 1 describes the background and related work. Section 2 discusses proposed method, Section 3 covers results & discussion and Section 4 concludes this paper.

2. Proposed method

Ultrasound images are corrupted with speckle noise. As this noise is multiplicative therefore its components get multiplied with the components of an image. Speckle noise follows gamma distribution [22]. Suppose input noisy image is I then

$$I = U \times N \tag{1}$$

where U is the noise free image and N is speckle noise. Image I has dimensions $M \times N$. Take log of (1) to convert multiplicative noise into additive noise. In this paper, before applying denoising filter, we will first classify each pixel of I into different image regions namely homogeneous, detail and edge. Each pixel is then convolved with fractional integral denoising mask to remove noise.

2.1. Image region classifier

Three different regions of an image (homogeneous, detail and edge) have distinct characteristics. Pixels of these regions can be differentiated on the basis of change in intensity value. This change is well reflected with the help of derivative. Hessian matrix is a 2×2 matrix based on second order derivative. Hessian matrix on each pixel of I will be defined as:

$$H_p = \begin{bmatrix} p_{xx} & p_{xy} \\ p_{yx} & p_{yy} \end{bmatrix} \tag{2}$$

where each element of matrix H is second order derivative of current pixel with its neighboring pixels, x and y represent x-direction and y-direction. $p \in M \times N$ which shows that Hessian matrix will be calculated for each pixel of I. If two neighbouring image pixels belong to the same image region, their second order derivative will be a small value. Pixels of different regions will have large value for second order derivative.

Hessian matrix has two eigenvalues:

$$\lambda_1 = \frac{1}{2}\left[(p_{xx} + p_{yy}) + \sqrt{(p_{xx} + p_{yy})^2 + 4p_{xy}^2} \right] \tag{3}$$

$$\lambda_2 = \frac{1}{2}\left[(p_{xx} + p_{yy}) - \sqrt{(p_{xx} + p_{yy})^2 + 4p_{xy}^2} \right] \tag{4}$$

where λ_1 and λ_2 are large and small eigenvalues respectively. p_{xx}, p_{yy} and p_{xy} are partial derivatives in x, y and xy directions.

When a pixel belongs to an edge, value of λ_1 is large and λ_2 is small. In homogeneous region, both λ_1 and λ_2 have very small values. So keeping this in mind, we can divide an image into different regions by considering only large eigenvalue (λ_1) of every pixel. But the critical thing is defining boundaries between different image regions. Apply the concept of rough set theory [24] on the values of λ_1 to classify each pixel into different image regions. Theory of rough set is an ideal tool for vague and overlapped concepts. This makes it suitable for handling speckle noise [23]. In a rough set, each class is divided into lower and upper approximations. *Lower* approximation corresponds to those elements of a universe which are certainly classified as a member of that class, *Upper* approximations is the set of all those elements which can possibly belong to a specific class [23]. In this paper, input image is the universe and λ_1 calculated for every pixel are the elements of this universe. These elements are divided into two classes namely *Small* and *Large* as shown in Fig. 1. Each rough class is further divided into *upper* and *lower* approximations. Pixels belonging to lower approximation of *Small* class will be those pixels which belong to homogeneous region, pixels in lower approximation of *Large* class belong to edge region and pixels which do not belong to any lower approximation are those pixels which belong to texture region. So in this way we can classify a pixel by considering large eigenvalue of its hessian matrix.