Computers and Electrical Engineering 000 (2016) 1–16

Contents lists available at ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

Multiresolution convex variational model for multiphase image segmentation ☆

Jiangxiong Fang [a,b,d,*], Hesheng Liu [c,*], Huaxiang Liu [c], Liting Zhang [a], Jun Liu [d], Huaiqiang Zhang [d], Congxin Liu [e]

[a] Jiangxi Province Key Lab for Digital Land, East China University of Technology, Nanchang, 330013, China
[b] Key Laboratory of Watershed Ecology and Geographical Environment Monitoring, NASG, Nanchang, 330013, China
[c] School of Mechanical and Electrical Engineering, East China University of Technology, Nanchang, 330013, China
[d] School of Nuclear Engineering and Geophysics, East China University of Technology, Nanchang, 330013, China
[e] The 771st Institute, China Aerospace Science and Technology Group, Xi'an, 710000, China

A R T I C L E I N F O

Article history:
Received 19 November 2015
Revised 24 April 2016
Accepted 25 April 2016
Available online xxx

Keywords:
Image segmentation
Convex formulation
Global minimization
Variational model

A B S T R A C T

This paper presents a convex variational model for multiphase image segmentation by incorporating a multiresolution approach. We extend our previous work to formulate the energy functional which is robust response to image variations. In contrast to our previous work, which can lead to local minima, a global solution is proposed to minimize the segmentation energy with some constraint conditions. By incorporating edge-based information, a non-convex energy functional is first introduced on the membership functions, which are used as indicators of different homogeneous regions. Then the non-convex problem is converted into a continuous convex formulation. An efficient dual minimization implementation of our binary partitioning function model accurately describes disjoint regions using stable segmentation. Experiments results show the proposed model is robust to noise, independent of initialization and unambiguous segmentation. Compared with the traditional variational models, the proposed model can get more accurate results and higher computational efficiency.

1. Introduction

Object segmentation is a key problem in the fields of image processing and computer vision. It plays an important role in numerous useful applications [1,6], since it facilitates the extraction of objects and interpretation of image contents. Over these years, many methods have been proposed to address the problem. Researchers have also made great efforts to improve the performance of the image segmentation algorithms. However, it is still difficult to segment the complicated images.

Active contour model is one of the most successful methods for image segmentation due to its well-grounded theory and flexibility. The well-known Chan–Vese model [3], which is derived from the Mumford–Shah model [2], is well used to segment an image with distinct mean of pixel intensity. With no reliance on the gradient to stop the propagation process, the model becomes an energy minimizing segmentation which can be seen as a particular case of the minimal partition

☆ Reviews processed and recommended for publication to the Editor-in-Chief by Area Editor Dr. E. Cabal-Yepez.

* Corresponding authors.
E-mail addresses: fangchj2002@163.com (J. Fang), hsliu@vip.163.com (H. Liu).

http://dx.doi.org/10.1016/j.compeleceng.2016.04.019
0045-7906/© 2016 Published by Elsevier Ltd.

problem. But in the process of curve evolution, the model needs to update the level set function by nonlinear partial differential equations (PDE) which cost more computation.

To improve its efficiency, a piecewise constant level set method [4] (PCLSM) was proposed to deal with multi-class classification by representing any number of phases. Multiphase level set method [5] has been proposed to segment the image with many regions. However, these models have the problems of vacuum and overlap due to each phase represented via one level set function. To solve these problems, the region competition model [1,4] is proposed by using $N-1$ level set functions to represent N regions. However, a main drawback of these models is the existence of local minima due to the non-convexity of the energy functional, which can't make sure that the algorithm can obtain the same results with different initialization.

Recently optimization techniques for active contour model become the focus. In these models [6–11], a convex formulation based on energy minimization is proposed to alleviate the problem of local minima. The classical convex approach [6,7] is proposed to segment the two-phase image. The energy functional of the method is globally minimized obtained using characteristic functions varying in the interval [0, 1]. This allows the non-convex Chan-Vese problem to be solved using standard convex optimization methods. A completely convex method [8,9] based on the Potts model [12] is proposed to tackle the multi-label problem by computing a global minimizer under certain conditions. Moreno et al. [10] proposed a binary partitioning formulation accurately described disjoint regions using dual minimization. A graph cut [11] with binary label-based super-level set functions is proposed to minimize the energy functional. However, these models do not aim at computing a global minimizer of the problem, so one cannot guarantee to find exact global minimizers of these models.

Inspired by Moreno et al. [10] and Bresson et al. [6], we extend the previous work in [15], whose energy functional has the advantages of accurate boundaries and robustness to image variations, to obtain a global solution by minimizing the multiphase segmentation energy with some constraint conditions and incorporating the multiresolution approach [16]. The main contributions of this paper are summarized as follows:

(1) By deriving an approximate convex functional we converted our previous formulation [15] into a binary segmentation problem and utilize a dual minimization [17] to solve the relaxed formulation. Moreover, we have proved the existence of a global minimizer in a specific space. The proposed global methodology avoids the level set re-initialization constraint.
(2) We apply the coarse-to-fine multiresolution schema [16] to improve the efficiency and increase the segmentation robustness to noise. Different from the multiresolution techniques [22] by decomposing the image into different resolutions using wavelet transform, we apply a multiresolution approach by coarse-to-fine downsampling in order to reduce the search space and provide a less noisy image due to detailed features being lost in the low resolution image.

The remainder of this paper is organized as follows. We first review related work in Section 2. In Section 3, we consider the global minimization of the multiphase segmentation model, including non-convex energy functional representation, convex formulation via energy functional lifting, dual formulation for energy minimization, and the algorithm for multiphase image segmentation. Experimental results illustrating the performances of segmentation are discussed in Section 4. Finally concluding remarks and suggestions for future works are given in Section 5.

2. Related work

Let an image domain Ω be divided into $\boldsymbol{N}$ non-overlapping regions Ω_i, $\boldsymbol{x}$ be a spatial variable $x \in \Omega$, the Potts model [12] is described by minimizing the constrained energy

$$\min_{\{\Omega_i\}_{i=1}^N} \sum_{i=1}^N \int_{\Omega_i} f_i(x)dx + \alpha \sum_{i=1}^N |\partial\Omega_i| \qquad \text{s.t.} \quad \cup_{i=1}^N \Omega_i = \Omega, \Omega_m \cap \Omega_n = \emptyset \quad \text{if} \quad m \neq n \tag{1}$$

where $|\partial\Omega_i|$ stands for the perimeter of the boundary of Ω_i, and $\alpha > 0$ is an arbitrary positive parameter. The first term is called the data term, and each $f_i(x)$ should depend on the input image $I(x)$. For example, $f_i(x) = |I(x) - c_i|^b$, $b = 1, 2$ represents the pixels that are classified in terms of the intensity means $\{c_i\}_{i=1}^N$. The second term, namely the regularization term, measures the sum of the perimeters of the sets Ω_i, i=1, ..., N. The functional (1) becomes the energy of the Mumford–Shah model [2] when the parameter $b = 2$:

$$\min_{(\{\Omega_i\}_{i=1}^N, \{c_i\}_{i=1}^N)} \sum_{i=1}^N \int_{\Omega_i} |I(x) - c_i|^2 dx + \alpha \sum_{i=1}^N |\partial\Omega_i| \tag{2}$$

By bringing an indicator function $\psi(x) = (\psi_1(x), \cdots, \psi_N(x))$ satisfying: $\psi_i(x) = 1$ if $x \in \Omega_i$ and $\psi_i(x) = 0$ if $x \notin \Omega_i$, the perimeter of each disjoint sub-domain is computed by $|\partial\Omega_i| = \frac{1}{2}\int_\Omega |\nabla\psi(x)|dx$. The Potts model reads

$$\min_{\psi_i \in [0,1]} \left\{ \sum_{i=1}^N \int_{\Omega_i} f_i(x)\psi_i(x)dx + \frac{\alpha}{2} \sum_{i=1}^N \int_\Omega |\nabla\psi_i(x)|dx \right\} \tag{3}$$