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Multiscale variational decomposition and its application for image hierarchical restoration ☆

Liming Tang [a,*], Chuanjiang He [b]

[a] School of Science, Hubei University for Nationalities, Enshi 445000, PR China
[b] College of Mathematics and Statistics, Chongqing University, Chongqing 401331, PR China

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ABSTRACT

Variational decomposition has been widely used in image denoising, however, it can't distinguish texture from noise well. Replacing the fixed parameter in the (BV, G) decomposition with a monotone increasing sequence, and iteratively taking the residual of the previous step as the input to decompose, we propose a multiscale variational decomposition model in this paper. Unlike the fixed-scale decomposition, the new model can decompose the input image into a sum of a series of features with different scales. So, texture can be distinguished from noise. In addition, we prove the nontrivial property and the convergence of this multiscale decomposition, and introduce a hybrid iteration algorithm that combines the first-order primal–dual algorithm with the gradient decent method to numerically solve the multiscale decomposition model. Numerical results validate the effectiveness of the proposed model. Furthermore, we apply this multiscale decomposition for image hierarchical restoration. Compared with the classical hierarchical (BV, L^2) decomposition, hierarchical wavelet decomposition and fixed-scale (BV, G) decomposition, our model has better performance for both synthetic and real images in terms of PSNR and MSSIM.

1. Introduction

Restoration of the true image from an observation is an important task in image processing, where the observed image is usually a noisy version of the true image. Given an image function $f(x,y) : \Omega \to \mathbb{R}$, image restoration is to extract true image u from the observation f. This is actually a decomposition of f, $f = u + v$, where u represents cartoon or structure component formed by piecewise smooth regions with sharp boundaries, and v represents oscillatory component consisting of scale repeated details. In order to achieve the decomposition in denoising case, one of the most well known techniques is by regularization and functional minimization, i.e., variational methods; see for example [1–9].

Total variation (TV) minimization model proposed by Rudin, Osher and Fatemi (ROF) [5] is a celebrated variational decomposition, in which an image $f \in L^2(\Omega)$ is split into the sum of $u \in BV(\Omega)$ and $v \in L^2(\Omega)$:

$$(u, v) = \arg\inf \left\{ |u|_{BV(\Omega)} + \mu \|v\|_{L^2(\Omega)}, \quad f = u + v \right\}; \tag{1.1}$$

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* Corresponding author. Tel.: +86 0718 8280564.
E-mail address: tlmcs78@foxmail.com (L. Tang).

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therefore it is also called $\left(BV, L^2\right)$ decomposition. Here $|u|_{BV(\Omega)}$ (also called TV of u) is a regularizing term to remove the noise. Model (1.1) is convex and easy to solve in practice. In addition, the function $u \in BV(\Omega)$ allows for discontinuities along curves, therefore edges and contours can be preserved in the restoration u.

However model (1.1) has some limitations. For instance, the model does not represent well oscillatory details such as textures since the oscillatory functions don't have small L^2-norm [7], and BV pieces are often sent to component v for any μ [10,11]. Meyer [7] first suggested using weaker norms for the oscillatory component instead of L^2-norm. One of his choices is the use of the dual norm of $BV(\Omega)$ for the oscillatory component. However, there is no known integral representation of continuous linear functional on $BV(\Omega)$. To address this problem, Meyer used another slightly larger space $G(\Omega) = W^{-1,\infty}(\Omega)$ to approximate the dual of $BV(\Omega)$, and introduced the following (BV, G) variational decomposition model,

$$\inf_{u \in BV(\Omega), v \in G(\Omega)} \left\{ |u|_{BV(\Omega)} + \mu \| v \|_{G(\Omega)},\ f = u + v \right\}. \tag{1.2}$$

In theory, this decomposition model can effectively extract oscillatory components from the observation. However, it cannot be directly solved in practice because there is no standard calculation of the associated Euler–Lagrange equation for energy in (1.2) [1,10–13].

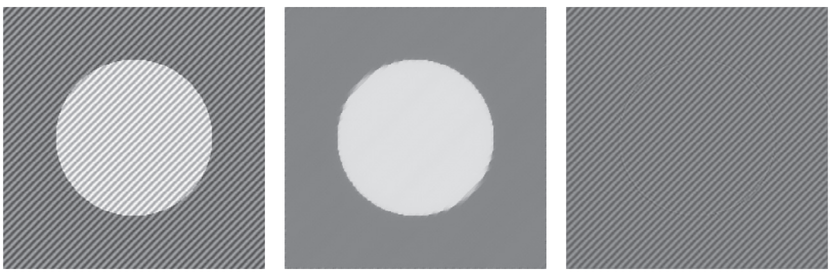
Vese and Osher [10,11] first overcame this difficulty by replacing the space $G(\Omega)$ with $G_p(\Omega) = W^{-1,p}(\Omega)$ $(1 \leqslant p < +\infty)$. Later on, in [14], these two authors together with Sole proposed an alternative approximation for the case of $p = 2$ in $G_p(\Omega)$, which coincides with the functional space $H^{-1}(\Omega)$ (the dual of the Hilbert space $H^1_0(\Omega)$). Aujol, Aubert, Blanc-Feraud and Chambolle (AABC) [12,15] proposed another approach in the dual framework to solve the Meyer's original (BV, G) model. Fig. 1 shows the result of (BV, G) decomposition for a synthetic image using AABC model. One can clearly see that G functional space can model well repeated pattern. Almost all textures are absorbed by oscillatory component v, but very few cartoon pieces are absorbed.

The (BV, G) model and it's approximations mentioned above are fixed-scale decomposition, since the tuning parameters in these models are fixed. It has been proved that a human visualizing a scene is in multiple scales [16], and image restoration from a noisy observation is actually a simulation of the human visual perception (HVP), i.e., extracting large-scale objects and contours from noisy observation, and simultaneously removing small-scale details. So, in order to identify meaningful features (e.g., contours and textures) and meaningless features (e.g., noise) under different scales, multiscale approaches simulating HVP are appropriate for image restoration and reconstruction. In addition, all these fixed-scale decomposition models split image features on a single-scale. so they cannot distinguish well textures and noises which have different scales. Beside that, searching the optimal parameters in these variational models is still a difficult task.

With the work of Tadmor, Nezzar and Vese in [17,18], in this paper we propose a multiscale (BV, G) decomposition model by replacing the fixed parameter in the original (BV, G) decomposition with a monotone increasing sequence, and iteratively taking the residual of the previous step decomposition as the input to decompose. New model can decompose the input image into a series of features with different scales. It the following applications and advantages:

- Our model can provide a multiscale representation of the input image in the framework of variation.
- We use the multiscale (BV, G) decomposition to restore the true image hierarchically, which can distinguish texture from noise more subtly than the corresponding fixed-scale decomposition.
- As a by-product, our multiscale (BV, G) decomposition can accurately extract texture under different scales, which is actually an active area of image processing [6,16,19,20].
- Using the proposed multiscale (BV, G) decomposition in denoising application, we do not need complex optimal parameter determination.

We perform extensive experiments on the proposed multiscale (BV, G) decomposition model and it's application for image hierarchical restoration. In addition, We compare their results with the classical hierarchical (BV, L^2) decomposition



(a) Test image (b) Cartoon u (c) Texture v

Fig. 1. The result of (BV, G) decomposition for a synthetic image.

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