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# Novel model of non-uniformly sampled-data systems based on a time-varying backward shift operator☆

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ABSTRACT

Based on the lifting technique, the lifted state space model and the lifted transfer function model can be derived to describe non-uniformly sampled-data (NUSD) systems. However, the lifted models are inconvenient for both identification and control purposes due to the causality constraint and the model complexity. To solve this problem, a novel model of NUSD systems is proposed by introducing a time-varying backward shift operator. The proposed model is more concise in structure and has fewer parameters than the lifted models, using which the traditional identification methods and control strategies of single-rate systems can be easily extended to NUSD systems. The advantages and effectiveness of the proposed model are well illustrated by a simulation example.



## 1. Introduction

Traditional identification methods and control strategies often assume that all the system inputs and outputs are synchronously sampled at the same rate. However, due to sensor limitations or data acquisition and transmission problems, some exceptions are encountered in practice [1]. For example, quality variables such as product concentration in distillation columns or melt index in polymerization units, etc., are infrequently obtained through off-line laboratory analyses, while flow, temperature, pressure, and other process variables can be online measured at relatively fast rates [2–4]. Such systems are known as multirate (MR) sampled-data systems and characterized by coexistence of two or more sampling rates.

MR systems have received extensive attentions in the fields of filtering, identification and control, and abundant achievements have been established [5,6]. The so-called lifting technique is a milestone in the area of MR systems, which is originated from the switch decomposition method proposed by Kranc [7]. Based on this technique, the MR system can be described as a single-rate state space model with higher dimension (i.e., the lifted state space model), enabling the MR problem to be studied under a single-rate framework [8]. However, the lifting technique can cause a causality constraint problem, which must be dealt with carefully in the identification and control of MR systems [9]. To avoid this problem, the lifted state space model can be transformed into the equivalent lifted transfer function model, but with that the number of model parameters is multiplied [10].

Non-uniformly sampled-data (NUSD) systems with irregular sampling intervals for the inputs and/or outputs are a class of general MR systems, which can be roughly classified into the following three categories:

(1) *NUSD systems with irregular inputs*
By applying the lifting technique to such NUSD systems, it is trivial to obtain the corresponding lifted state space models or the lifted transfer function models. For the NUSD systems with white noise interference, a gradient based iterative algorithm has been derived to estimate the parameters of the lifted output error (OE) model [11]. To reduce the computation complexity of the conventional recursive

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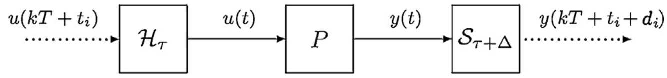


**Fig. 1.** Non-uniformly sampled-data systems with asynchronous input–output data.

least squares (RLS) algorithm, a hierarchical least squares identification algorithm for the lifted auto-regressive model with exogenous input was presented in [12].

Under the circumstance with colored noise interference, a filtering based RLS algorithm was developed to identify the lifted Box–Jenkins model [13]. Taking into consideration of the prior knowledge on the model parameters and the noises, a recursive Bayesian algorithm with covariance resetting was proposed in [14]. Furthermore, the identification of Wiener nonlinear systems with non-uniformly updated inputs was discussed and a gradient-based iterative algorithm was provided in [15].

(2) *NUSD systems with irregular outputs*

In process industries, the quality variables often require manual sampling and laboratory analyses; thus their sampling is scarce and non-uniformly spaced, leading to NUSD systems with irregular outputs. Such NUSD systems are widely studied especially in the domains of soft sensor and inferential control [16,17]. Moreover, the conventional single-rate systems with randomly missing outputs can also be viewed as such NUSD systems [18,19]. Besides, the intentional nonuniform sampling of the outputs is often employed in the networked control systems to reduce the network congestion [20,21].

For the identification of linear systems with missing data, Raghavan et al. combined the expectation-maximization (EM) algorithm with a Kalman filter to estimate the parameters of the fast-rate state space model [22]. Furthermore, the EM algorithm in combination with particle filter is presented to identify nonlinear systems with irregular missing observations [18]. However, the EM-based identification algorithms require large computational cost and must be iteratively performed in an off-line manner. Recently, Tulsyan et al. derived an online Bayesian approach to simultaneously estimate the states and the parameters of nonlinear systems with missing measurements [23].

(3) *NUSD systems with irregular inputs and outputs*

In the case of synchronous input updating and output sampling, Ding et al. proposed a hierarchical identification algorithm for the lifted state space model, and further studied the reconstruction of the original continuous-time system [24]. Also, a modified subspace identification algorithm was presented in [25], where the causality constraint was tackled by decomposition of the lifted measurement equation. Moreover, a partially coupled stochastic gradient algorithm [26] and an auxiliary model based multi-innovation generalized extended stochastic gradient algorithm [27] were proposed for identification of the lifted transfer function model.

Assuming that the non-uniform sampling interval is small and uniformly bounded, Yuz et al. utilized the EM algorithm to identify the continuous-time systems [28]. Furthermore, Chen et al. studied identification of continuous-time systems with arbitrary time-delay based on the irregularly sampled input–output data [29]. For more complex NUSD systems with asynchronous input updating and output sampling, Li et al. proposed a novel subspace approach to identify the lifted state space model, and further investigated the problem of fault detection and isolation [30,31]. Considering random delays associated with the external input-to-filter and the output-to-filter, Liu et al. studied $H_2$ and $H_\infty$ filtering for NUSD systems with asynchronous input–output data [32].

To the best of our knowledge, many identification and control methods for NUSD systems are proposed based on the lifted state space models or the lifted transfer function models. However, these two models both have their own limitations, i.e., the former suffers from the problem of causality constraint, and the latter is complex and includes a large number of parameters. To overcome the limitations of the lifted models, this paper proposes a novel input–output representation of NUSD systems by introducing a time-varying backward shift operator. The major advantage of the proposed model lies in its concise structure with fewer parameters.

The rest of this paper is organized as follows. Section 2 presents the problem formulation, followed by the model derivation in Section 3. Sections 4 and 5 discuss the identification and control of NUSD systems based on the proposed novel model, respectively. Section 6 provides an illustration example. Finally, conclusions are given in Section 7.

## 2. Problem formulation

Consider a class of NUSD systems with asynchronous input–output data as depicted in Fig. 1, where $P$ is a continuous linear time-invariant (LTI) process described by the following state space representation:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t), \\ y(t) = \boldsymbol{C}\boldsymbol{x}(t) + Du(t). \end{cases} \tag{1}$$

Here, $u(t)$ is the process input generated by passing the discrete-time input sequence $u(kT+t_i)$ through a non-uniform zero-order hold $\mathcal{H}_\tau$; $y(t)$ is the process output sampled by a non-uniform sampler $\mathcal{S}_{\tau+\Delta}$, yielding the output sequence $y(kT+t_i+d_i)$; $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector; $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ are constant matrices with appropriate dimensions, and $D$ is a constant.

Furthermore, $\mathcal{H}_\tau$ and $\mathcal{S}_{\tau+\Delta}$ are assumed to have a periodic non-uniform updating and sampling pattern as illustrated in Fig. 2. The discrete-time inputs $\{u(kT+t_i)\}$ are updated $r$ times within each frame period $T$, and the sampling instants $\{kT+t_i, i=0, 1, 2, \ldots, r-1\}$ are non-uniformly spaced with intervals $\{\tau_1, \tau_2, \ldots, \tau_r\}$, where $t_0=0$, $t_i=t_{i-1}+\tau_i$ for $i=1, 2, \ldots, r$, $T=t_r=\sum_{i=1}^{r}\tau_i$. The outputs $\{y(kT+t_i+d_i)\}$ are sampled at time instants $\{kT+t_i+d_i\}$, which are not synchronized with the inputs and there exist time lags $d_i \in [0, \tau_{i+1})$, $i=0, 1, 2, \ldots, r-1$. When $d_i=0$, the asynchronous NUSD systems are reduced to synchronous ones as studied in [25,26].

Due to the non-uniform and asynchronous input updating and output sampling, the mapping relationship between the sampled inputs and outputs becomes nonlinear. Therefore, it is necessary to derive discrete-time models of NUSD systems for the effective identification and control objectives.