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# Robust discrete-time set-based adaptive predictive control for nonlinear systems

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ABSTRACT

The problem of robust adaptive predictive control for a class of discrete-time nonlinear systems is considered. First, a parameter estimation technique, based on an uncertainty set estimation, is formulated. This technique is able to provide robust performance for nonlinear systems subject to exogenous variables. Second, an adaptive MPC is developed to use the uncertainty estimation in a framework of min–max robust control. A Lipschitz-based approach, which provides a conservative approximation for the min–max problem, is used to solve the control problem, retaining the computational complexity of nominal MPC formulations and the robustness of the min–max approach. Finally, the set-based estimation algorithm and the robust predictive controller are successfully applied in two case studies. The first one is the control of anonisothermal CSTR governed by the van de Vusse reaction. Concentration and temperature regulation is considered with the simultaneous estimation of the frequency (or pre-exponential) factors of the Arrhenius equation. In the second example, a biomedical model for chemotherapy control is simulated using control actions provided by the proposed algorithm. The methods for estimation and control were tested using different disturbances scenarios.



## 1. Introduction

Model predictive control (MPC) has experienced considerable attention in the academic literature in the last two decades. Moreover, it has also been largely applied by industries, due to its ability to enforce constraints and handle multivariable process effectively and efficiently. However, the presence of uncertainty in the MPC problem formulation remains a challenging topic. The presence of uncertainty requires feedback and optimization over a sequence of control laws rather than optimization over sequences of control actions, as in nominal MPC [19]. Despite academic effort in the design of robust nonlinear model predictive control (NMPC) systems, the problems associated with parametric uncertainties remains a considerable challenge in applications. The presence of parametric uncertainties can have severe implications in the implementation of reliable NMPC systems. In classical control, this task can be handled using a vast array of adaptive control and adaptive estimation techniques. The situation in MPC poses some additional challenges. The main problem with the application of an adaptive control approach in NMPC systems is that the uncertain parameters may impact the quality of the model predictions drastically and, hence, the performance of the control system. It is therefore imperative that the NMPC approach preserves robustness to parametric uncertainties while taking full advantage of the potential NMPC performance gains.

The problem of using measurements for online update of model parameters in MPC (so called adaptive MPC) has received some attention in the literature. In general, the model and the uncertainty descriptions of a robust MPC are configured for a nominal set of operating conditions that are typically not updated. The nominal parameter uncertainties are thus lumped with other structured or unstructured uncertainty descriptions which yields conservative robust control systems. Parametric uncertainty is usually handled by imposing bounds on the unknown parameters. Various robust MPC mechanism can then be employed to mitigate their impact on the model predictions and the MPC system. Min–max robust MPC approaches can be used to handle such parametric uncertainties (see [23] and the references therein). The conservatism associated with such approaches can be overcome if one is able to anticipate the effect of future changes in the uncertainties. In the case of adaptive MPC, the objective is to use online learning algorithms that use plant measurements to update the uncertain parameters. Such algorithms are usually equipped with some guarantees of convergence of the parameters in a way that can be used to forecast

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their impact on future model predictions. For linear systems some results are available. In [10], an adaptive approach is considered where an exponential decay for parametric uncertainty is used in the model prediction. An interesting result was proposed in [17] where a persistency of excitation condition is used to prove that robust feasibility is preserved if no states constraints are used.

In a recent study [26], a set-membership adaptive MPC approach was proposed. In this technique, a class of linear systems represented by impulse response coefficients convolution models is considered. Under the assumption that the model of the uncertain plant belongs to a class of linear systems with bounded impulse response coefficients, a set-membership update strategy is used to identify models that are consistent with past input-output responses of the uncertain linear system. The technique is shown to provide accurate set membership assignment and guarantees robust performance of the unknown linear systems. In the context of the current study, the approach proposes to update the parameters following a model set-membership approach following an empirical model approach. Furthermore, it is limited to open-loop stable systems, with the possibility of integrating behaviour. A similar approach is proposed in [6] where a set membership identification technique due to [20] is used to identify nonlinear systems from a class of Lipschitz nonlinear operators based on the closeness of the process data. Again, the approach is limited to open-loop stable systems. The current study provides a robust adaptive stabilization result for a class of uncertain nonlinear systems. The parameterization is assumed to be known but the uncertainty in the parameters can be effectively updated in real-time to minimize the impact of the uncertain parameters.

For the adaptive MPC control of nonlinear systems, some authors have proposed the use of adaptive neural network models (for example see [5,4,24]). For nonlinear constrained systems, [1] proposed a robust framework for continuous time systems, in which the transient effect of parameter estimation error was explicitly used in the robust control problem. In [3], the previous work was extended for continuous systems with disturbances. Finally, an adaptive robust economic MPC, based on the results found in [1,3], was proposed in [14].

Another framework to design an adaptive predictive controller considers the use of Kalman Filters and Moving Horizon Estimation (MHE) to obtain estimates of the parameters. In this approach, the state vector is augmented with the unknown parameter values under the assumption of a constant parameter vector [25]. In the context of nonlinear adaptive MPC, this technique was used by [9] for joint estimation using an Extended Kalman Filter (EKF) for a polymerization reactor. A combination of MHE and MPC in the adaptive framework is shown in [8], where the optimal dosing of cancer chemotherapy problem is addressed. This approach can be applied to a large class of nonlinear systems. However, the dynamic uncertainty estimation for application in a robust MPC problem is not a trivial problem. Moreover, the computation of the parameter estimates using an MHE approach increases the computational cost, since two nonlinear optimization problems should be solved sequentially. The presented solution proposes the use of an algorithm that allows the dynamic uncertainty set update and preserves the computational cost of usual nonlinear MPC, leading to an implementable robust algorithm.

In this work, we establish a theoretical basis for the analysis of robust adaptive MPC control system subject to exogenous disturbances for a class of discrete-time nonlinear control systems. The result generalizes the continuous-time approach first proposed in [1]. No claims are made concerning the computational requirements of the proposed min–max approach to the adaptive MPC technique. However, it is argued that a Lipschitz-based approach provides a conservative approximation of the min–max approach that retains all of the stability and robustness properties. The uncertainties associated with the parameters are handled using a new set-based estimation approach for a class of nonlinear discrete-time systems that guarantees contraction of the uncertainty set in the presence of a persistency of excitation condition. Moreover, it is shown how this set-based approach can be formulated in the context of nonlinear adaptive MPC approach for discrete-time systems in the presence of parameter uncertainties and exogenous disturbances.

The remainder of the paper is structured as follows. The problem description is given in Section 2. The parameter estimation routine is presented in Section 3. Two approaches to robust adaptive model predictive control are detailed in Section 4. This is followed by a simulation example in Section 6 and brief conclusions in Section 7.

## 2. Problem description

Consider the uncertain discrete-time nonlinear system:

$$x_{k+1} = x_k + F(x_k, u_k) + G(x_k, u_k)\theta + \vartheta_k \triangleq \mathcal{F}(x_k, u_k, \theta, \vartheta_k) \tag{1}$$

where the disturbance $\vartheta_k \in \mathcal{D} \subset \mathbb{R}^{n_d}$ is assumed to satisfy a known upper bound $||\vartheta_k|| \leq M_\vartheta < \infty$. The objective of the study is to (robustly) stabilize the plant to some target set $\Xi \subset \mathbb{R}^{n_x}$ while satisfying the point-wise constraints $x_k \in \mathcal{X} \in \mathbb{R}^{n_x}$ and $u_k \in \mathcal{U} \in \mathbb{R}^{n_u}$, $\forall k \in \mathbb{Z}$. The target set is a compact set, contains the origin and is robustly invariant under no control. It is assumed that $\theta$ is uniquely identifiable and lies within an initially known compact set $\Theta^0 = B(\theta_0, z_\theta)$ where $\theta_0$ is a nominal parameter value, $z_\theta$ is the radius of the parameter uncertainty set.

**Remark 1.** In this study, the exogenous variable $\vartheta_k$ represents an unstructured bounded time-varying uncertainty. We do not provide any additional structure, such as a state dependent disturbance matrix, since this is assumed to be expressed by the term $G(x_k, u_k)\theta$ in (1).

## 3. Parameter and uncertainty set estimation

In this section, we present and analyze the proposed set-based parameter estimation technique.

### 3.1. Parameter adaptation

The preferred parameter estimation technique is first presented. The main idea behind the proposed technique is the definition of an implicit regression model. The implicit model is based on the definition of a vector of auxiliary variables, denoted by $\eta_k$. The dynamics of $\eta_k$ forms the basis for the proposed estimation approach.

The first element required is filtered form of the regressor vector $G(x_k, u_k)$ denoted by $\omega_k$. The vector $\omega_k$ is obtained using the following recursion:

$$\omega_{k+1} = \omega_k + G(x_k, u_k) - K_k\omega_k, \quad \omega_0 = 0 \tag{2}$$

where $K_k$ is a correction factor at time step $k$. Note that $\omega_k$ has the same dimension as $G(x_k, u_k)$.

We let $\hat{\theta}_k$ be the vector of parameter estimates at time step $k$. Using the process model and the filter (2), we propose the following state predictor:

$$\begin{aligned}\hat{x}_{k+1} = {} & \hat{x}_k + F(x_k, u_k) + G(x_k, u_k)\hat{\theta}_{k+1} + K_k e_k - \omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}) \\ & + K_k\omega_k(\hat{\theta}_k - \hat{\theta}_{k+1})\end{aligned} \tag{3}$$

where $e_k = x_k - \hat{x}_k$ is the state estimation error at time step $k$. The state predictor is used to generate information about the parameter estimates.