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A nonparametric approach to design robust controllers for uncertain systems: Application to an air flow heating system

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ABSTRACT

This paper presents an approach to design robust fixed structure controllers for uncertain systems using a finite set of measurements in the frequency domain. In traditional control system design, usually, based on measurements, a model of the plant, which is only an approximation of the physical system, is first built, and then control approaches are used to design a controller based on the identified model. Errors associated with the identification process as well as the inevitable uncertainties associated with plant parameter variations, external disturbances, measurement noise, etc. are expected to all contribute to the degradation of the performance of such a scheme. In this paper, we propose a nonparametric method that uses frequency-domain data to directly design a robust controller, for a class of uncertainties, without the need for model identification. The proposed technique, which is based on interval analysis, allows us to take into account the plant uncertainties during the controller synthesis itself. The technique relies on computing the controller parameters for which the set of all possible frequency responses of the closed-loop system are included in the envelope of a desired frequency response. Such an inclusion problem can be solved using interval techniques. The main advantages of the proposed approach are: (1) the control design does not require any mathematical model, (2) the controller is robust with respect to plant uncertainties, and (3) the controller structure can be chosen *a priori*, which allows us to select low-order controllers. To illustrate the proposed method and demonstrate its efficacy, an application to an air flow heating system is presented.

1. Introduction

Development of models and control laws for complex systems requires accurate knowledge about the plant dynamics. Mathematical descriptions of such complex systems can be obtained using physical laws. However, the use of physical laws usually leads to a high-order set of algebraic or differential equations which can be difficult to obtain and solve, and this may make the control design task more challenging. To this end, most control approaches available in the literature are based on mathematical models extracted from data [1–4]. The derivation of such models are usually based on some prior assumptions such as structure, order and time delay of the model, which are often unavailable or subject to uncertainties. Hence, errors associated with such identified models may result in deterioration of the closed-loop system performance [5,6]. An accurate description of the plant dynamics can usually be obtained from measured input–output data. A possible alternative that circumvents problems regarding parametric identification of the plant is to directly utilize measured data for the controller design process. Such measured data can be given either in the time-domain or frequency-domain. The authors in [7–11] used time-domain data to design controllers for reference and tracking problems. In the frequency domain, data-based controller tuning is often based on the frequency response of the plant which is assumed to be available or can be measured experimentally [12–18]. Such an assumption is often valid in many practical applications. It has been shown in [12,13] that the set of all stabilizing PID controllers achieving some desired gain and phase margins or infinity-norm constraints can be obtained using frequency-domain data. A technique to design data-based controllers ensuring

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performance specifications either in terms of gain and phase margins or in terms of the infinity-norm of weighted sensitivity and complementary sensitivity functions, has been presented in [14,15]. Based on plant frequency response data, the authors in [16,17] proposed a method to design controllers that minimize the upper and lower bounds of the infinity-norm of weighted sensitivity and complementary sensitivity functions. A data-based control design method has been presented in [18], where the parameters of a pre-selected controller structure are optimized with respect to the closed-loop H_∞ performance. The aforementioned data-based control approaches are based on a set of measurements carried out on the open-loop system (plant); however, some research related to data-based controller design using closed-loop measured data is also available. Such control techniques based on closed-loop data are very useful in the control of unstable plants. For instance, the authors in [19] proposed an approach to design an LQG controller using a set of closed-loop measurements. A closed-loop scheme to design data-based controllers with guaranteed stability has been presented in [20]. The authors in [21] developed a new modeling technique for unknown complex systems, which has also been used to design controllers using closed-loop frequency domain data. Data-based approaches for designing controllers with guaranteed bounded error have recently been proposed in [22,23]. Although such frequency-domain data-based approaches are advantageous in terms of avoiding the errors associated with the system model, they are sensitive to measurement noise and environmental changes. Moreover, real systems often exhibit strong variation in their physical parameters. These latter have a significant impact on the accuracy of the resulting frequency response of the plant. Therefore, control design methods that are based on inaccurate frequency response data would have a profound impact on the desired closed-loop performance or even closed-loop stability. Hence, uncertainties related to the plant frequency response should be taken into account during the controller synthesis. Depending on the application, uncertainties can be described in different ways, such as affine forms, stochastic distributions, etc. Interval analysis is one way to handle a class of uncertainties, and provides a powerful mathematical tool that allows us to characterize some uncertain quantity simply by its upper and lower bounds. Interval uncertainty description is a simple and natural way to bound the uncertainty in the plant frequency response by intervals. The information about the upper and lower bounds of the uncertain frequency response of the plant can then be used for control design purposes. The control design is based on combining the principles of interval arithmetics with those of linear control theory. In addition to the simplicity of this concept, one main advantage is that low-order controllers can easily be derived without the use of any interval mathematical model. The first idea of using interval arithmetic goes back to the work of Burkill [24] and Young, [25], and then later in 1966 to R.E. Moore's work [26]. The tools developed in these works have found use in different areas, especially in the field of systems and control, such as: modeling [27,32], stability analysis [28,33–35] and control design [27–32,36–38]. The book [27] deals with robust stability analysis and fixed-order controller design ensuring robust performance for uncertain systems. A chapter in the book [28] is dedicated to the use of interval arithmetic for robust stability analysis and design of robust stabilizing controllers for parametric uncertain systems. It has been shown in [29–31] that several problems of stability analysis and robust design of control systems in the presence of parametric uncertainties can easily be solved using interval analysis algorithms. The authors in [32] developed a method in the frequency domain to design robust controllers achieving gain and phase margins for interval systems. A robust control design approach for interval systems has been presented in [36], where two controllers were designed: a feedback stabilizing controller is first computed, then a pre-compensator is designed to meet the desired performance specifications. Recently, a robust control design approach based on the inclusion test of interval parameters of transfer functions has been proposed in [37]. This latter has been applied to control piezoelectric actuators. In [38], interval arithmetic has been combined with H_∞-technique to design robust controller for uncertain systems. The results in all of the papers cited above are based on the use of transfer functions with interval parameters. In this paper, we propose a nonparametric robust control method using only the plant frequency response template without going through the use of any interval transfer function. Contrarily to data-based control design approaches outlined beforehand, where controllers are designed for linear time-invariant systems with no uncertainty, our attempt in this paper is to consider the problem of data-based control design for systems associated with uncertainty and disturbances due mainly to environmental changes and physical parameter variation. The underlying concept is naturally very simple, thanks to the availability of interval analysis.

The paper is structured as follows. Section 2 provides some useful mathematical preliminaries relating to interval theory and uncertain plant frequency response template. In Section 3, we present a detailed formulation and solution of the robust control design problem. Section 4 is dedicated to a demonstration of the efficacy of the proposed method through a practical example. Finally, Section 5 provides some concluding remarks.

2. Classical definitions on interval analysis and uncertain plant frequency response

2.1. Basic terms and concepts for intervals

In this subsection, some mathematical preliminaries for real and complex intervals are presented. More details on these preliminaries can be found in [28,39,40].

Definition 1. A closed real interval denoted by $[x]$, is the set of real numbers given by:

$$[x] = [\underline{x}, \overline{x}] = \left\{ x \in \mathbb{R} : \underline{x} \le x \le \overline{x} \right\}. \quad (1)$$

where the endpoints $\underline{x}$ and $\overline{x}$ are respectively the left and right endpoints of $[x]$.

Table 1 gives some classical mathematical operations on two intervals $[x] = [\underline{x}, \overline{x}]$ and $[y] = [\underline{y}, \overline{y}]$.

Definition 2. A complex interval $[z]$ in the complex plane (rectangular form) is the set of complex numbers defined as follows [40,41]:

$$\begin{aligned}[z] &= [z_r] + j[z_i] \\ &= \left\{ (z_r + jz_i) \in \mathbb{C} : \underline{z_r} \le z_r \le \overline{z_r} \text{and} \underline{z_i} \le z_i \le \overline{z_i} \right\},\end{aligned} \quad (2)$$

where $j^2 = -1$. $[z_r]$ and $[z_i]$ are two real intervals that represent the real and imaginary parts of the complex interval $[z]$. $\underline{z_r}$ and $\overline{z_r}$ (resp. $\underline{z_i}$ and $\overline{z_i}$) are the endpoints of the interval $[z_r]$ (resp. $[z_i]$).

It is important to note that the basic mathematical operations on complex intervals are similar to the operations on real interval

Table 1
Classical arithmetic operations on intervals [28,39].

Operation	Definition
+	$[x] + [y] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$
−	$[x] - [y] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$
*	$[x] * [y] = [\min\{\underline{x} * \underline{y}, \overline{x} * \underline{y}, \underline{x} * \overline{y}, \overline{x} * \overline{y}\}, \max\{\underline{x} * \underline{y}, \overline{x} * \underline{y}, \underline{x} * \overline{y}, \overline{x} * \overline{y}\}]$
/	$[x]/[y] = [x] * [1/\overline{y}, 1/\underline{y}], 0 \notin [y]$