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Stabilizing model predictive control using parameter-dependent dynamic policy for nonlinear systems modeled with neural networks

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ABSTRACT

A class of parameter-dependent dynamic control policies is explored for its use in a model predictive control (MPC) algorithm for a nonlinear system modeled with a feedforward neural network (NN). The NN-modeled system is expressed as a polytopic quasi-linear-parameter-varying (quasi-LPV) system over a region of the state-input space for a range of operating points, and the dynamics of the proposed policy, which are optimized off-line to enlarge the region of attraction, are allowed to depend on a time-varying parameter of the polytopic quasi-LPV system model such that the resulting control involves a continuous gain-scheduling that leads to reduced conservativeness. A complete MPC algorithm using the dynamic policy as the terminal policy ensures stabilization and improved control performance over a larger domain of attraction without a larger horizon length. Simulation examples with tank and tubular reactor systems illustrate the effective performance of the proposed approach in practical applications.

1. Introduction

Model predictive control (MPC) is often regarded as one of the most effective techniques available for the control of constrained systems in the process industries. The technique uses a model of plant dynamics to optimize control inputs in real time based on the predicted plant behavior [1,2]. However, for many practical systems, which may be nonlinear, an exact mathematical model of the plant may be difficult, if not impossible, to derive or express analytically in a simple, discrete-time form. One way of modeling such a system is to use a suitable neural network (NN) to represent the system dynamics. NN-based modeling has been considered in several earlier works such as [3–9], and different control approaches have been explored for the resulting system. While nonlinearities may be satisfactorily modeled using NNs, the resulting MPC problem is usually significantly more complex than a linear MPC problem.

Solving a nonlinear MPC problem with a guarantee of stability is usually computationally demanding. The traditional finite-horizon nonlinear MPC usually requires a sufficiently large horizon length to ensure a reliable performance [10]. Computationally attractive alternatives have been proposed over the past several years [11–16] and, relying on the fact that a nonlinear system can be modeled as a quasi-linear-parameter-varying (quasi-LPV) system[1] so that its trajectories can be embedded into a linear difference inclusion [[17], Section 4.3], some of these proposals (e.g., [12,13,15]) have used approaches inspired by those designed for the robust-MPC-and/or gain-scheduling-based solution to the control problem for LPV systems, such as [18,19]. The resulting control ensures stability but the solution is usually suboptimal and, in many cases, is specific to a chosen fixed operating point. Nonlinear MPC schemes that are applicable for the tracking of a family of reference points have been explored in a number of earlier works (e.g., [20–23]). The authors in [20] have employed a pseudo-linearization of the system to transform it into a constant linear system for all the desired operating points and considered a quasi-infinite-horizon MPC for the transformed system. [21] has presented a scheme using an online switching between a set of locally stabilizing robust static controllers with overlapping regions of attraction. In [22], a dynamic controller that is stabilizing for all the desired operating conditions is assumed to exist and used as the terminal controller appended to the standard MPC law. A more general, and possibly less conservative, approach employing an auxiliary steady state as an additional

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[1] An LPV system is referred to as a quasi-LPV system if the parameters depend on the system state and/or input.

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decision variable and a terminal set for the state-reference combination is presented in [23], which actually extends the results for linear systems in [24]. Most of these schemes (e.g., [20,22,23]) envisage the existence and the use of a terminal control policy that offers a terminal region of attraction in order to ensure stability. [20] also presents a method, albeit one that appears to be a bit cumbersome, for the computation of the terminal controller and its region of attraction, and [23] discusses various options including the approach of [21] as the candidates for the terminal controller. In the recent years, off-line-optimized dynamic-controller-based MPC policies have been proposed for a class of (uncertain) linear systems [25–27] and they are found to be computationally attractive and also suitable for their use as the terminal control law (e.g., [27,28]). Clearly, an optimized dynamic control policy in line with policies proposed for linear systems can reduce the conservativeness and improve the online computational efficiency of a nonlinear MPC scheme.

In this paper, we explore an optimized-dynamic-policy-based stabilizing MPC scheme to solve the nonlinear tracking problem for a system whose dynamics may be partly unknown and modeled with a feedforward NN (ff-NN). Some preliminary studies in this direction have been made in [29]. Here, we explore the problem in a more general setting in which the operating point may not be fixed at the time of design. We train a NN to represent the plant dynamics for a set of operating points and obtain a polytopic quasi-LPV description of the NN-modeled system that is applicable for any operating point in the chosen set. For this description, we design a dynamic policy that depends on the time-varying parameter of the LPV model and hence offers a less conservative domain of attraction. However, since the usual dynamic policies in [25,26], etc. are designed with an underlying assumption that the parameter is uncertain, these policies allow only the predicted future control inputs to depend on the value of the parameter, thus resulting in a kind of 'virtual' gain scheduling. In the context of a quasi-LPV system, the parameter may not be really uncertain. Therefore, we propose a more general form of the policy with controller dynamics parameterized quadratically in terms of the time-varying parameter. Such a parameterization allows an actual gain-scheduling such as in [30] and usually results in a less conservative domain of attraction and possibly a better control performance. We also incorporate, in the policy dynamics, approximation errors arising from the NN-based modeling to avoid conservativeness due to such errors. Finally, when the dynamic policy is used as the terminal policy in the MPC formulation, since it enlarges the stabilizing terminal set while also allowing an optimization of the terminal input sequence, the overall MPC solution can be expected to offer stability and optimality over a larger domain without a larger horizon length.

We present a successive-quadratic-programming (succ-QP)-based algorithm to solve the overall nonlinear MPC problem for the NN-modeled system. Some earlier works on finite-horizon MPC for NN-modeled systems such as [3,8] have used successive substitution/optimization methods and obtained feasible solutions considering the nonlinear part as an additional constant disturbance. However, such solutions are only approximate, and offer no guarantee of feasibility and stability. The algorithm in this paper handles the dynamics-related nonlinear constraints in the finite horizon by imposing linearized versions of the constraints and employs the proposed dynamic policy as the stabilizing terminal policy. The finite-horizon part of the algorithm resembles the approach discussed in [31,32] and also employed in different variants in other papers such as [9]. Since the analytical expression for the output of the NN allows an efficient online linearization, the proposed algorithm offers an improved performance, together with a guarantee of stability in an efficient way. We assess and illustrate the performance of the proposed approach with several numerical examples including those dealing with the tank and tubular reactors. The example with a tubular reactor presents the case of an MPC-oriented modeling and effective control of a distributed parameter system.

Notations: $I(I_n)$ denotes an identity matrix (of size n) and $\mathbf{1}$ denotes a vector of all ones. $\lceil a \rceil$ represents the smallest integer larger than a. For vectors x and y, $(x\,;y)$ represents the stacked vector $[x^T\,y^T]^T$. For a vector x, diag(x) denotes a diagonal matrix with the components of x along its diagonal. For a matrix X, $X_{[i,:]}$ denotes its ith row and $X_{[i,j]}$ denotes its element on the ith row and jth column. For matrices X and Y, $X \otimes Y$ denotes their Kronecker product, and diag(X,Y) represents a block diagonal matrix with X and Y as blocks. The signs $\succeq$, $\prec$ etc. denote positive/negative (semi-) definiteness of matrices. $\mathbb{Z}_+$ and $\mathbb{R}_+$ represent the sets of non-negative integers and real numbers respectively. For two sets U and V, $U \oplus V$ denotes their sum, $U \ominus V$ denotes the difference, $U \cup V$ denotes their union and $U \backslash V$ represents the complement of V in U. Co denotes a convex hull. A convex and compact set with the origin in its interior is denoted as a $\mathcal{C}$-set.

2. Problem description

We consider a plant described by dynamics of the form

$$x^a(t+1) = f(x^a(t), u^a(t)) \tag{1}$$

where $x^a(t) \in \mathbb{R}^{n_x}$ and $u^a(t) \in \mathbb{R}^{n_u}$ represent the actual state of the plant and the control input applied. The function $f(.\,,.)$ is supposed to be continuous but it may not be fully known or may be difficult to be expressed analytically in a simple form.

We assume that it is known through simulations, experiments or analyses that the system can be steadily maintained at any point x^o in a set $\mathcal{X}^o$ with a respective control input $u^o \in \mathcal{U}^o$ so that an appropriately defined system output $y^a(t) = g(x^a, u^a)$ can be maintained at the respective set-point $r^a \in \mathcal{R}$. The control objective at any time t is to drive the system to such a steady state. The system state and the control input are supposed to satisfy the constraints $x^a(t) \in \mathbb{X}^a$ and $u^a(t) \in \mathbb{U}^a$, where $\mathbb{X}^a$ and $\mathbb{U}^a$ are $\mathcal{C}$-sets.

Given a set-point $r^a \in \mathcal{R}$ and the corresponding steady state-input pair (x^o, u^o), we wish to compute, at each time instant t, a sequence of predicted control inputs $u^a(t+i|t)$, $i \in \mathbb{Z}_+$ that minimize a cost function of the form

$$J(t) = \sum_{i=0}^{\infty} \left\{ \left\| Q^{\frac{1}{2}}(x^a(t+i|t) - x^o) \right\|^2 + \left\| R^{\frac{1}{2}}\,(u^a(t+i|t) - u^o) \right\|^2 \right\}, \quad Q \succ 0, R \succ 0 \tag{2}$$

so that the closed-loop system state under the receding horizon control $u^{a^*}(t) = u^{a^*}(t|t)$ is driven to and maintained at the point x^o in some optimal way.

Defining $x(t) = x^a(t) - x^o$ and $u(t) = u^a(t) - u^o$, we note that the problem of driving the system to a steady state-input pair (x^o, u^o) is equivalent to steering $(x(t), u(t))$ to the origin. In the following sections, we discuss an appropriate modeling of dynamics of $x(t)$ with a suitable ff-NN and then present an MPC procedure based on the resulting nonlinear model. We assume that it is possible to obtain a sufficiently rich set of input/output data from experiments or simulations such that the dynamics can be modeled with a NN of an appropriate structure.