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Functional diagnosability and detectability of nonlinear models based on analytical redundancy relations

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ABSTRACT

This paper introduces an original definition of diagnosability for nonlinear dynamical models called *functional diagnosability*. Fault diagnosability characterizes the faults that can be discriminated using the available sensors in a system. The functional diagnosability definition proposed in this paper is based on analytical redundancy relations obtained from differential algebra tools. Contrary to classical definitions, the study of functional diagnosability highlights some of the analytical redundancy relations properties related to the fault acting on the system. Additionally, it gives a criterion for detecting the faults. Interestingly, the proposed diagnosability definition is closely linked to the notion of identifiability, which establishes an unambiguous mapping between the parameters and the output trajectories of a model. This link allows us to provide a sufficient condition for testing functional diagnosability of a system. Numerical simulations attest the relevance of the suggested approach.

1. Introduction

Fault diagnosability establishes which faults can be discriminated according to the available sensors in a system. By analyzing diagnosability, it is possible to anticipate the discriminatory power of a diagnoser at run time and to propose solutions to other important problems like the one of selecting the lowest cardinality sensor set that guarantees discriminability of an anticipated set of faults. Diagnosability analysis must be achieved in the framework used to design the diagnoser, which is in our case the model-based framework. The principle of model-based fault diagnosis is to compare the behavior of the system with the predictions that arise from the model and to analyse the sources of discrepancy. In the case of nonlinear models, the classical methods are based on nonlinear observers ([13] for example) and/or analytical redundancy relations (ARRs) [14,15]. These latter are relations linking the system inputs, outputs and their derivatives. This paper follows the second track and proposes an extension of the existing definitions and methods for diagnosability and detectability from ARRs. The extension is in line with a gain of discriminability.

The considered nonlinear dynamical parameterized models (controlled or uncontrolled) are of the following form:

$$\begin{cases} \dot{x}(t,p,f) = g(x(t,p),u(t),f,\varepsilon(t),p), \\ y(t,p,f) = h(x(t,p),u(t),f,\varepsilon(t),p), \\ x(t_0,p,f) = x_0, \\ t_0 \le t \le T. \end{cases} \tag{1}$$

where

- $x(t,p,f) \in \mathbb{R}^n$ and $y(t,p,f) \in \mathbb{R}^m$ denote the state variables and the outputs respectively,
- the functions g and h are real, rational and analytic on M, where M is an open set of $\mathbb{R}^n$ such that $x(t,p,f) \in M$ for every $t \in [t_0, T]$. T is a finite or infinite time bound,
- $u(t) \in \mathbb{R}^r$ is the control vector,
- $f \in \mathbb{R}^e$ is the fault vector,
- $\varepsilon(t)$ is a stochastic vector introducing noise in the system,
- the vector of parameters p belongs to $\mathcal{U}_{\mathcal{P}}$, where $\mathcal{U}_{\mathcal{P}} \subseteq \mathbb{R}^q$ is an a priori known set of admissible parameters,
- the initial conditions x_0 are assumed to belong to a bounded set $\mathcal{X}_0$, to be independent of f and to be different from an equilibrium point of the system.

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$f = 0$ means no fault and $\varepsilon = 0$ means no noise. In the case of uncontrolled models $u = 0$.

From elimination theory, some differential polynomials, also called *input–output representations*, that may act as ARRs – since they link system inputs, outputs, parameters and their derivatives – can be obtained. In the last decade, algorithms for obtaining such ARRs have been developed and implemented in softwares as Maple [1]. They are based on differential algebra [6] and allow one to eliminate state variables, which are unknown, from the model. ARRs can be used to detect [5], isolate and estimate faults or in other words to achieve fault detection and isolation (FDI) [17]. To do so, a so-called *residual* is associated to each ARR, and acts as a consistency indicator [14].

In our paper, faults are considered to disturb the system model (1). Interestingly, there is no restriction about the type of faults. They may act multiplicatively changing the value of some parameter already present in the model or as additional parameters. FDI then relies on the assumption that the model parameterization is suitably chosen so that the faults of the system can be detected and isolated. The purpose of diagnosability analysis is to verify such property.

Some definitions of diagnosability based on ARRs have been proposed in the literature. A classical diagnosability definition stands in comparing fault signatures [15]. Typically, the fault signature of a fault is a Boolean vector referring to a set of residuals and reporting which residuals are sensitive (with a 1) and not sensitive (with a 0) to the fault. According to [15], the model is said diagnosable if for any two faults, their fault signatures are distinct. Then, if two faults act on the same residuals, the model is not diagnosable.

Ref. [2] considers that a system is diagnosable if f is algebraically observable with respect to u and y. Defining f_i as the ith component of the fault vector f, it means that each fault component f_i can be written as a solution of a polynomial equation in f_i and finitely many time derivatives of inputs u and outputs y. This definition can be likened to the definition of identifiability proposed in [10]. Indeed, the parameters are defined globally identifiable if the condition above stands for each parameter p_i. Considering the fault vector as a parameter vector, classical identifiability and diagnosability as proposed by [2] are hence equivalent. The links between the notions of identifiability and diagnosability and the correspondence between faults and parameters have actually been sensed by several authors among which those of [17]. Their work is based on the key paper [10] that presents a method based on the use of input–output representations – from which ARRs can be built – for studying the identifiability of a model. In [10], input–output representations are obtained with the Ritt's algorithm and checking identifiability may require a lot of manipulations of the model equations. As a result, it is often impossible to obtain such input–output representations for complex systems and, if they are obtained, the order of derivatives is so high that the relations cannot be used as ARRs for FDI, thus the limitation of the method proposed by [2] for diagnosability analysis and FDI. Aware of these problems, [17] relaxes the condition required by global identifiability, and allows the input–output representations to involve several parameters/faults. Input–output representations are used to build residuals evaluated thanks to a statistical change detection method. Hence [17] proposes an FDI method for non linear systems but does not consider diagnosability analysis and the problem of providing conditions for faults to be discriminable.

In [3], thanks to the Rosenfeld–Groebner algorithm, which is by far more efficient than the Ritt's algorithm [1], and to a particular elimination order, the authors propose to study the identifiability of the parameters of a model from differential polynomials that may contain more than one parameters. The advantage of these polynomials is that they present a particular form allowing one to provide general conditions to study identifiability. Furthermore, they contain derivatives of lower order than the ones required by [10]. We borrow the idea of [3] for diagnosability analysis and propose a new definition of diagnosability, called *functional diagnosability*. This definition is closely related to the classical definition of identifiability. From this link, definitions of fault detectability and discriminability are proposed and a sufficient condition for verifying functional diagnosability is deduced. The method proposed to verify this condition is easy to implement and allows to detect faults whereas traditional methods fail, as illustrated by the example of the water tanks presented in Section 4.2.

The paper is organized as follows. In Section 2, a general method for obtaining specific ARRs is presented. In Section 3, the definition of functional diagnosability is introduced and linked to the notion of identifiability. From this study, a criterion is given for testing functional diagnosability. Section 4 presents two numerical examples and Section 5 discusses the results and concludes the paper.

2. Obtention of ARRs

In the following subsections, the expression of ARRs and how to obtain ARRs through variable elimination are presented.

2.1. ARRs and their decomposition

In [14], the authors propose to use ARRs for fault detection and isolation in algebraic dynamic systems. An ARR is a relation deduced from the model of the system that links the system inputs and outputs and their derivatives. Provided that derivatives can be estimated, an ARR is hence a testable relation in the sense that it can be evaluated with the measurements and this is why it is useful in the FDI framework.

The following notations borrowed from [14] are used. If ϑ is a vector, $\bar{\vartheta}^{(k)}$ is the vector whose components are ϑ and its time derivatives up to order k, $\bar{\vartheta}$ stands for ϑ and its time derivatives up to some (unspecified) order. Consider the set of ARRs:

$$w_i(\bar{y}, \bar{u}, f, \bar{\varepsilon}, p) = 0, \quad i = 1, \ldots, m. \tag{2}$$

They can be decomposed as:

$$w_i(\bar{y}, \bar{u}, f, \bar{\varepsilon}, p) = w_{d,i}(\bar{y}, \bar{u}, f, p) - w_{s,i}(\bar{y}, \bar{u}, f, \bar{\varepsilon}, p) = 0, \tag{3}$$

where $w_{d,i}(\bar{y}, \bar{u}, f, p)$ is the deterministic part (a polynomial of degree zero in the components of $\bar{\varepsilon}$) and $w_{s,i}(\bar{y}, \bar{u}, f, \bar{\varepsilon}, p)$ is the stochastic part (a polynomial of degree at least one in some components of $\bar{\varepsilon}$).

In most cases, there is no simple characterization of the residual's stochastic behavior, in particular for established fault detection procedures. [17] provides an FDI method that perfectly exemplifies how stochastic aspects can be managed. However, other papers like [14] propose to base fault detection on the deterministic part of the residual and we also adopt this assumption. With this assumption, $w_{s,i}(\bar{y}, \bar{u}, f, \bar{\varepsilon}, p) = 0$ and ARRs can be rewritten:

$$w_i(\bar{y}, \bar{u}, f, \bar{\varepsilon}, p) = w_{d,i}(\bar{y}, \bar{u}, f, p) = 0. \tag{4}$$

$w_{d,i}(\bar{y}, \bar{u}, f, p)$ can be decomposed as:

$$w_{d,i}(\bar{y}, \bar{u}, f, p) = w_{0,i}(\bar{y}, \bar{u}, p) - w_{1,i}(\bar{y}, \bar{u}, f, p), \tag{5}$$

where $w_{0,i}(\bar{y}, \bar{u}, p)$ is a fault-free term and $w_{1,i}(\bar{y}, \bar{u}, f, p)$ is a term that depends on the fault vector. Consequently:

$$w_i(\bar{y}, \bar{u}, f, p) = w_{0,i}(\bar{y}, \bar{u}, p) - w_{1,i}(\bar{y}, \bar{u}, f, p). \tag{6}$$

According to (4), the following relation is always true:

$$w_{0,i}(\bar{y}, \bar{u}, p) = w_{1,i}(\bar{y}, \bar{u}, f, p). \tag{7}$$