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A bias correction method for fractional closed-loop system identification

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ABSTRACT

In this paper, the fractional closed-loop system identification using the indirect approach is presented. A bias correction method is developed to deal with the bias problem in the continuous-time fractional closed-loop system identification. This method is based on the least squares estimator combined with the state variable filter approach. The basic idea is to eliminate the estimation bias by adding a correction term in the least squares estimates. The proposed algorithm is extended, using a nonlinear optimization algorithm, to estimate both coefficients and commensurate-order of the process. Numerical example shows the performances of the fractional order bias eliminated least squares method via Monte Carlo simulations.

1. Introduction

Recently, several researches show that the fractional calculus provides an excellent tool to describe the behavior of many complex physical systems due essentially to their long memory characteristic. This induces the use of fractional models in many applications involving various theoretical fields such as control [1,2], diagnosis [3] and system identification [4,5].

Concerning the time-domain fractional system identification, it has received great interest in the late nineties. In Cois et al. work [6] the fractional differentiation orders are assumed to be known *a priori* and only the coefficients are estimated by minimizing the quadratic criterion based on the equation-error. A synthesis of fractional Laguerre basis for system identification is developed in [7]. An optimal instrumental variable method for continuous-time fractional system identification has been proposed in [8]. In Chetoui et al. work [4] a new methods based on higher-order statistics are illustrated for the estimation of both fractional orders and coefficients of continuous-time errors-in-variables fractional models. In the bounded error context, Amairi et al. works extended several set-membership methods to the fractional system identification such as the outer bounding ellipsoid (OBE) [9] and the outer bounding parallelotope (OBP) [10].

The methods mentioned above are developed in open-loop conditions. However, for many industrial production process, the experimental data can only be obtained in closed-loop conditions for several reasons like stability, security, safety and performance constraints. In the literature, three approaches are proposed for the closed-loop system identification: the direct approach, the indirect approach and the joint input/output approach [11].

As for rational systems, (see [12,11,13] and the references therein for more details) the fractional closed-loop system identification has also attracted an attention recently. In [14], an unstable fractional first-order system with input time-delay has been identified using a graphical method based on the step response of the fractional closed-loop system. A more general indirect approach has been proposed in [15] where the least squares algorithm combined with the state variable filter is used. Simulation results have shown the presence of a bias on the estimates when the system is contaminated by a high level additive white noise. To solve this problem, the bias eliminated least squares (*bels*) method proposed in [16] is extended to the fractional case in [17]. This method is called the fractional order bias eliminated least squares (*fobels*).

The *fobels* method is composed by three major steps. In the first step, the least squares method is used to estimate the closed-loop parameters. In the second step, the bias introduced by this method is estimated by an appropriate algorithm. The third step consists in computing the process parameters using the bias correction

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principle. The algorithm proposed in [17], besides its efficiency, it requires a restriction on the controller order. To remove this restriction, like as the rational case (see [18] and [19] for more details), we propose in this paper an extended version of the *fobels* method called the prefileterd *fobels* (*pfobels*) which deals with an arbitrary order controller. The idea is to use a stable prefilter with an appropriate order connected to the external excitation of the closed-loop system.

This paper is organized as follows: a mathematical background of fractional systems is presented in Section 2. The problem statement is illustrated in Section 3. In Section 4, the extension of the bias eliminated least squares method to the fractional closed-loop systems is developed. In Section 5, the algorithm of the prefiltered *fobels* method is described. Both coefficients and fractional orders are estimated in the Section 6. In Section 7, a numerical example shows the performances of the developed method. Finally, Section 8 concludes this paper.

2. Mathematical background

Several definitions of the fractional differentiation have been proposed in the literature [20–22]. In this paper only the Grünwald-Letnikov definition is used [23].

Definition 1. The υ−Grünwald fractional differentiation of a continuous-time function *f*(*t*) relaxed at $t=0$ (i.e. $f(t)=0, \forall t \le 0$) is defined by

$$D^{\upsilon} f(t) \simeq \frac{1}{h^{\upsilon}} \sum_{k=0}^{K} (-1)^k \binom{\upsilon}{k} f(t-kh), \quad \forall t \in \mathbb{R}_+^* \tag{1}$$

$\mathbb{R}_+^*$ denotes the set of all strictly positive real numbers. $\mathrm{D} = \left(\frac{d}{dt}\right)$ is the time domain differential operator, h is the sampling period, $t = Kh$ and $\binom{\upsilon}{k}$ is the Newton's binomial generalized to fractional orders such as

$$\binom{\upsilon}{k} = \begin{cases} 1 & \text{if } k=0 \\ \dfrac{\upsilon(\upsilon-1)(\upsilon-2)\ldots(\upsilon-k+1)}{k!} & \text{if } k>0 \end{cases} \tag{2}$$

Consider a SISO commensurate-order[1] fractional system described by

$$\sum_{i=0}^{n_a} a_i \mathrm{D}^{i\upsilon} y(t) = \sum_{j=0}^{n_b} b_j \mathrm{D}^{j\upsilon} y_c(t) \tag{3}$$

where $(a_i, b_j) \in \mathbb{R}^2$ are the linear coefficients of the differential equation, $\upsilon \in \mathbb{R}_+^*$ is the commensurate-order and where $y_c(t)$ and $y(t)$ are respectively the input and the output signals.

Applying the Laplace transform to the fractional differential Eq. (3), under zero initial conditions, yields the fractional transfer function

$$G(s) = \frac{B(s)}{A(s)} = \frac{\sum_{j=0}^{n_b} b_j s^{j\upsilon}}{\sum_{i=0}^{n_a} a_i s^{i\upsilon}} \tag{4}$$

Stability condition for the fractional systems has been established in [21].

Theorem 1. *A commensurate-order system described by (4) is Bounded Input Bonded Output (BIBO) stable iff*

$$0 < \upsilon < 2 \tag{5}$$

and

$$\forall s_k^{\upsilon} \in \mathbb{C}, \quad A\left(s_k^{\upsilon}\right) = 0 \quad \text{such that} \quad \left|\arg\left(s_k^{\upsilon}\right)\right| > \upsilon \frac{\pi}{2} \tag{6}$$

where s_k^{υ} is a pole of the commensurate transfer function.

[1] All differentiation orders are exactly divisible by the same number, an integral number of times.

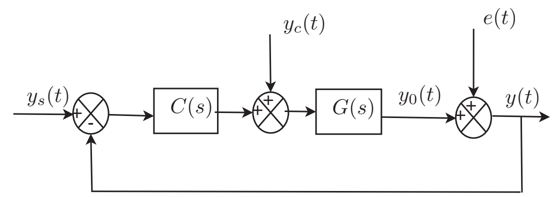

Fig. 1. Fractional closed-loop system.

3. Problem statement

Consider the fractional closed-loop system depicted in Fig. 1.

$G(s)$ and $C(s)$ describe respectively the process transfer function and the controller transfer function. The signals $y_s(t)$, $y_c(t)$ and $y_0(t)$ are respectively the set-point input, the continuous-time noise-free input and the process output.

The measurable output signal $y(t)$ is eventually corrupted by an additive noise $e(t)$ such as

$$y(t) = y_0(t) + e(t) \tag{7}$$

The process transfer function is considered commensurate and given by

$$G(s) = \frac{B(s)}{A(s)} = \frac{\sum_{j=0}^{n_b} b_j s^{j\upsilon}}{\sum_{i=0}^{n_a} a_i s^{i\upsilon}}; \qquad n_a \ge n_b \tag{8}$$

where $(a_i, b_j) \in \mathbb{R}^2$ are the linear coefficients of the process transfer function and $\upsilon \in \mathbb{R}_+^*$ is the process commensurate-order.

The controller transfer function is assumed to be known and described by a commensurate transfer function as

$$C(s) = \frac{Q(s)}{P(s)} = \frac{\sum_{r=0}^{n_q} q_r s^{r\mu}}{\sum_{l=0}^{n_p} p_l s^{l\mu}}; \qquad n_p \ge n_q \tag{9}$$

where $(p_l, q_r) \in \mathbb{R}^2$ are the linear coefficients of the controller transfer function and $\mu \in \mathbb{R}_+^*$ is the controller commensurate-order.

In this paper, the input signal $y_c(t)$ is considered perfecty known and the output signal $y(t)$ is measured. The controller output (the control signal) is supposed unmeasurable. Thus, the indirect approach is required to identify the process with fractional model using *a prior* knowledge of the controller.

Due to the feedback control, there exist a correlation between the unmeasurable output noise and the control signals [11]. In the case where the process and the controller are represented respectively by (8) and (9), the correlation is amplified due to the long memory aspect of the fractional differentiation.

Recently, some works present contributions in the fractional closed-loop system identification context [15,17,24]. The simulation results presented in [15] have shown that for an important additive noise the estimated parameters are biased. To eliminate this bias, the optimal instrumental variable method combined with a nonlinear optimization algorithm is handled to identify both fractional transfer function coefficients and fractional orders [24]. An extension of the bias eliminated least squares (*bels*) method to fractional order case has been proposed in [17]. This method gives an unbiased estimation but, a restriction on the controller order is verified.

The objective of this paper is to extend the *bels* method to fractional order case to identify the fractional closed-loop systems without noise modelling and without any restriction on the controller order.