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Optimal design and operation of energy systems under uncertainty

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ABSTRACT

This paper is concerned with integrated design and operation of energy systems that are subject to significant uncertainties. The problem is cast as a two-stage stochastic programming problem, which can be transformed into a large-scale nonconvex mixed-integer nonlinear programming problem (MINLP). The MINLP exhibits a decomposable structure that can be exploited by nonconvex generalized Benders decomposition (NGBD) for efficient global optimization. This paper extends the NGBD method developed by the authors recently, such that the method can handle non-separable functions and integer operational decisions. Both the standard NGBD algorithm and an enhanced one with piecewise convex relaxations are discussed. The advantages of the proposed formulation and solution method are demonstrated through case studies of two industrial energy systems, a natural gas production network and a polygeneration plant. The first example shows that the two-stage stochastic programming formulation can result in better expected economic performance than the deterministic formulation, and that NGBD is more efficient than a state-of-the-art global optimization solver. The second example shows that the integration of piecewise convex relaxations can improve the efficiency of NGBD by at least an order of magnitude.

1. Introduction

Global primary energy demand is projected to increase by over one third from 2011 to 2035 [1]. This will result in increasing needs for developing new and expanding existing energy systems, especially clean and/or renewable energy systems (e.g. natural gas production systems, biofuel plants) due to concerns with energy and environmental sustainability. For the development of an energy system, a design problem is often considered together with an operational problem. In other words, the physical infrastructure and the conditions under which the system should be operated are to be determined simultaneously.

One challenge for integrated design and operation comes from the uncertainties in the system, i.e., the factors that are not known when the design problem is considered. The uncertain factors may be the raw material and product prices, raw material availability and quality, physical–chemical relationships, etc. On the other hand, these uncertain factors can often be realized or estimated with a high precision after the system is developed, so that the system operating conditions can be determined with a precise and deterministic model. Therefore, the uncertainties in an integrated design and operation problem can be addressed via the following two-stage stochastic programming framework:

$$\min_{y\in Y} h(y) + E_{\xi\in\Xi}\{Q(y,\xi)\}, \quad \text{(TSSP)}$$

where y denotes the design decisions, such as the topological structure of the system and capacities of the units, and $h(y)$ represents the total costs associated with y, typically the total investment costs of the system. ξ denotes the uncertain factors that are assumed to be unknown before the system is developed but known afterward, and Ξ denotes the set containing all possible realizations of ξ under consideration. $Q(y, \xi)$ represents the net cost induced by the operation of the system, typically the total operating cost minus the total revenue (i.e., the negative of the total profit), with a given design y and a particular realization of uncertainties ξ. Often the operation of the system with a given y and a known ξ is determined by solving an optimization problem; in this case, $Q(y, \xi)$ is the optimal value of the following optimization problem:

$$\begin{aligned} Q(y,\xi) = \; & \min_{x} q(x,y,\xi) \\ & \text{s.t. } g(x,y,\xi) \le 0, \\ & x \in X(\xi), \end{aligned} \quad \text{(RP)}$$

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where x denotes a set of variables relevant to the operating conditions of the system, such as pressures and temperatures, and $q(x, y, \xi)$ represents the net operating cost for a particular group of design/operational decisions and uncertainty realizations. In the stochastic programming literature, Problem (RP) is called a recourse problem [2].

If the set Ξ is infinite, Problem (TSSP) is generally intractable, as an infinite number of recourse problems are to be solved. A typical approach to solve Problem (TSSP) is to approximate the set Ξ with a finite subset $\tilde{\Xi} = \{\xi_1, \ldots, \xi_s\} \subset \Xi$. Each element of $\tilde{\Xi}$ is called a scenario. The probability of each scenario, α_ω, can often be estimated by known uncertainty distributions or historical data, and $\sum_{\omega=1}^{s} \alpha_\omega = 1$. With this approximation, only a finite number of recourse problems need to be addressed, and Problem (TSSP) can be reformulated in the following form:

$$\begin{aligned} \min_{x_1,\ldots,x_s,y} \ & \sum_{\omega=1}^{s} \alpha_\omega (h(y) + q(x_\omega, y, \xi_\omega)) \\ \text{s.t. } & g(x_\omega, y, \xi_\omega) \le 0, \quad \omega = 1, \ldots, s, \\ & x \in X(\xi_\omega), \quad \omega = 1, \ldots, s, \\ & y \in Y. \end{aligned} \tag{STSSP}$$

Problem (STSSP) represents the classical scenario formulation for addressing uncertainties. Another classical formulation for addressing uncertainties is a robust formulation, in which the cost of the worst-case scenario (instead of the expected cost of a finite of scenarios) is considered. The robust formulation can ensure feasibility of solution for the problem, but it cannot ensure optimality of the solution if the expected cost is to be minimized. In this paper, the robust formulation is not considered, as we assume that (a) the energy systems of consideration involve large investment costs and throughputs, so the inability to achieve the best expected economic performance may result in significant loss of profit; (b) feasibility of the solution for a finite number of scenarios is sufficient.

The other challenge for integrated design and operation comes from the need to determine different system operating conditions for different operating modes. For example, in order to achieve the best profit, a power plant may generate more electricity in the morning and less at night due to different electricity prices, or more electricity in weekdays and less at weekends due to different amounts of customer demand. In this case, the design problem needs to be integrated with multiple operational problems instead of a single one, and the integrated design and operation problem is to ensure that the designed system can work under the different operating conditions and that it achieves the best total profit over all the operating modes. This optimization problem can also be cast as a problem in the form of (STSSP); but in this case, set $\tilde{\Xi}$ includes the values of the deterministic parameters that may be different for different operating modes, instead of realizations of uncertain parameters, and the p_ω represents the frequencies of occurrence of the operating modes.

For convenience of subsequent discussions, Problem (STSSP) is expressed as follows:

$$\begin{aligned} \min_{x_1,\ldots,x_s,y} \ & \sum_{\omega=1}^{s} f_\omega(x_\omega, y) \\ \text{s.t. } & g_\omega(x_\omega, y) \le 0, \quad \omega = 1, \ldots, s, \\ & x_\omega \in X_\omega, \quad \omega = 1, \ldots, s, \\ & y \in Y, \end{aligned} \tag{P}$$

where sets $X_\omega = \{x_\omega = (x_{c,\omega}, x_{b,\omega}) \subset \mathbb{R}^{n_{x_c}} \times \{0,1\}^{n_{x_b}} : p_\omega(x_\omega) \le 0\}$, $Y \subset \{0,1\}^{n_y}$, functions $f_\omega : \mathbb{R}^{n_{x_c}} \times \{0,1\}^{n_{x_b}+n_y} \to \mathbb{R}$, $g_\omega : \mathbb{R}^{n_{x_c}} \times \{0,1\}^{n_{x_b}+n_y} \to \mathbb{R}^m$, $p_\omega : \mathbb{R}^{n_{x_c}} \times \{0,1\}^{n_{x_b}} \to \mathbb{R}^{m_p}$ are continuous. It is assumed that at least one function in the problem is nonconvex (which is often the case for energy system design and operation), then Problem (P) is a nonconvex mixed-integer nonlinear programming (MINLP) problem. The size of Problem (P) grows linearly with s (i.e., number of scenarios or operating modes considered); when s is large, the problem is a large-scale MINLP. While Problem (P) only addresses binary design decisions y, it can be used for problems with bounded integer design decisions, as any bounded integer variable can be represented by a linear combination of a finite number of binary variables.

Problem (P) needs to be solved to global optimality to achieve the highest total profit (or the lowest total cost), especially for systems involving large investment costs and high throughputs. However, the global optimization of the nonconvex MINLP is often computationally challenging, because the solution times of classical global optimization methods, such as branch-and-reduce ([3]), SMIN-αBB and GMIN-αBB ([4]), and nonconvex outer approximation ([5]), increase dramatically with the problem size.

This paper focuses on efficient global optimization of Problem (P) using nonconvex generalized Benders decomposition (NGBD), a novel global optimization method recently developed by the authors [6,7]. It is an extension of the authors' paper for DYCOPS 2013 [8], and compared to the previous paper, it includes the following new contents:

1. As given earlier in this section, it shows how the integrated energy system design and operation problem can be cast as a two-stage stochastic programming problem, and why a scenario approximation of this problem is a deterministic mixed-integer nonlinear programming (MINLP) problem with a decomposable structure.
2. It presents NGBD for a more general problem formulation, where the functions may not be separable in the first-stage variables y and the second-stage variables x, and x may involve both continuous and binary decisions. Due to this generalization, the article presents a slightly different NGBD method. It also includes additional discussion on how to reformulate functions involving binary variables, for generating valid lower bounding problems for NGBD.
3. Due to generalization of the problem formulation, both NGBD and the piecewise convex relaxation methods are described using a different notation system.

This paper is organized as follows: Section 2 gives a brief introduction to the basic NGBD method. Section 3 presents a piecewise convex relaxation framework for generating tighter bounds for NGBD, and Section 4 discusses the integration of this framework with NGBD. The benefit of the scenario formulation for addressing uncertainty and the computational advantage of NGBD with piecewise convex relaxations are demonstrated through two industrial problems in Section 5. Section 6 concludes the paper and gives suggestions for future work.

2. Nonconvex generalized benders decomposition

It is well known that Problem (P) can be solved by Benders decomposition (BD) [9] or generalized Benders decomposition (GBD) [10], if the functions in the problem are convex and the operational decisions are continuous (although separability is also usually necessary). In BD or GBD, the problem is solved