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Smooth switching in a scheduled robust model predictive controller

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ABSTRACT

This paper proposes a bumpless transfer method to overcome the problem of switching jumps in a scheduled robust model predictive control approach. A scheduled robust model predictive controller implements a set of local robust model predictive controllers based on an on-line switching strategy. This method could enlarge the domain of attraction efficiently but the transient response might be hampered by spikes appearing at the moment of switching between adjacent local controllers. The proposed algorithm could enhance the transient response by implementing some intermediate controllers augmented to the main control scheme to solve the problem without needing more computation. The efficiency of the proposed algorithm is illustrated with two examples.

1. Introduction

Industrial applications of model predictive controllers (MPC's) have increased monotonically in the recent years because of their ability to handle those systems that have constraints on their variables, interactions among the continuous and logical subsystems, and delays between inputs and outputs. These controllers provide guaranteed stability in the most practical cases especially when the system behaves almost linearly. However, they act inadequately on nonlinear systems because of the computational complexity which is important in on-line implementation of the controllers and also lacking a guaranteed stability analysis [1]. In this context, Kothare et al. [2] have proposed a robust model predictive controller (RMPC) for constrained nonlinear systems which could be represented by linear models with polytopic uncertainties. This method minimizes an upper bound on the worst case infinite horizon objective function subject to input and state constraints instead of solving a nonlinear optimization problem in nonlinear MPC (NMPC) content which generally involves high complexity. Moreover, the region of stability of this controller is specified explicitly. These advantages have provided an area with great potentials for further researches.

Meanwhile, some researchers investigated approaches to decrease computational time and complexity [3,4], reduce conservativeness [5–8], or simplify the representation of the uncertainties [9–12]. In [9] the nonlinear system was represented by a linear model along with Lipschitz bounds on the nonlinearity approximation error which could somehow simplify the model representation while increase the conservativeness of the designed controller. Also [10] used this idea to introduce another representation for RMPC formulation. Estimating the region of stability for nonlinear control systems is an important issue. Furthermore, finding approaches to enlarge this region is another problem of interest [13–15]. In this regard, Wan and Kothare [13] have proposed scheduled RMPC that could enhance the stability region efficiently. The proposed approach in [13] implements the family of local RMPC's for a set of equilibrium points with appropriate overlaps in their regions of stability. This scheme acts as a single scheduled RMPC that switches on-line between the local controllers and achieves nonlinear transitions with guaranteed stability. Similarly, [16] used linear quadratic regulators instead of RMPCs as local controllers in the scheduled structure where a control law regarding each local controller was computed off-line. As a result, the on-line computational burden is reduced however the performance may degrade substantially. Switching among multiple controllers in the scheduling control techniques may produce non-smooth transient responses for manipulating signals which could lead to mechanical damage, fatigue loading, or signal saturations. Therefore it is undesirable in practical applications where the bumpless transferring is an important issue [17,18].

In this paper the scheduling scheme presented in [13] is implemented on an RMPC designed based on [10] in order to cover more space in the operating region. Then the scheduling approach is modified to alleviate the jumping problem in the transient response occurs because of switching between multiple local controllers and

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enhances the overall control performance. In the proposed method the off-line computations is similar to that in the scheduled RMPC in [13], however a novel on-line switching strategy is implemented to smooth the transient response.

The rest of the paper is organized as follows. In Section 2, the intended modifications on the proposed RMPC approach in [10] are discussed. The modified scheduling RMPC is described in Section 3. Section 4 presents the proposed bumpless approach. Section 5 studies two numerical examples in order to demonstrate the effectiveness of the proposed method. Finally, conclusions are derived in Section 6.

2. Design of an RMPC

In this section an RMPC formulation based on the corrected version of method introduced in [10] is presented. Consider a discrete-time nonlinear system as

$$x(k+1) = f(x(k), u(k)), \quad (1)$$

where $u(k) \in \mathcal{R}^m$ is the control input subject to $|u_r(k)| \le u_{r,\max}$, $r = 1, 2, \ldots, m$, $x(k) \in \mathcal{R}^n$ is the state of the system subject to $|x_r(k)| \le x_{r,\max}$, $r = 1, 2, \ldots, n$, and $f(\cdot, \cdot)$ is considered a Lipschitz and C^1 function where $f(0,0) = 0$ (it is assumed that the origin is the equilibrium of the system). The nonlinear model in (1) can be reformulated as

$$x(k+1) = Ax(k) + Bu(k) + \tilde{f}(x(k), u(k)), \quad (2)$$

where $A = \partial f / \partial x\big|_{(0,0)}$, $B = \partial f / \partial u\big|_{(0,0)}$. Since f is a Lipschitz nonlinearity then $\tilde{f}(x(k), u(k)) = f(x(k), u(k)) - Ax(k) - Bu(k)$ is bounded as

$$\tilde{f}(x(k), u(k))^T \tilde{f}(x(k), u(k)) \le [x(k); u(k)]^T L^T L [x(k); u(k)], \quad (3)$$

To regulate the system to the origin as the main control goal, it is assumed that the pair (A,B) is stabilizable. The state feedback is found by using MPC. In MPC method a cost function is minimized to optimize performance of the closed loop system for a time horizon in the future. Usually the first computed control move is implemented. Then based on the new measurements obtained from the plant, the calculations are repeated at the next sampling times. To determine a state feedback control law for $u(\cdot)$ in system (1), the objective function is defined as follows

$$J(k) = \sum_{i=0}^{\infty} x(k+i|k)^T Q x(k+i|k) + u(k+i|k)^T R u(k+i|k), \quad (4)$$

where $Q > 0$, $R > 0$. $x(k+i|k)$ is state at time $k+i$ predicted based on the measurements at time k, $x(k)$, and $u(k+i|k)$ is control move at time $k+i$ computed by minimizing $J(k)$ at time k.

To solve the given optimization problem for the nonlinear system in (1) via LMI, first one should replace equality in (4) by an inequality which is done by defining an upper bound for $J(k)$. Suppose a quadratic function $V(x) = x^T P x$ with $P > 0$ and $V(0) = 0$ satisfies the following inequality at sampling time k

$$V(x(k+i+1|k)) - V(x(k+i|k)) \le -x(k+i|k)^T Q x(k+i|k) - u(k+i|k)^T R u(k+i|k). \quad (5)$$

By summing both sides of (5) from $i = 0$ to $i = \infty$ one finds that

$$x(\infty|k)^T P x(\infty|k) - x(k|k)^T P x(k|k) \le -J(k). \quad (6)$$

For the asymptotic stability of the closed loop system, $x(\infty|k)$ must be zero and thus to have an asymptotic stability it follows that

$$J(k) \le V(x(k|k)) \le \gamma, \quad (7)$$

where γ is a positive scalar and is an upper bound for (4). As a result the RMPC problem is defined as follows.

Theorem 1. *Consider system (1) subject to $\left|u_r(k+i|k)\right| \le u_{r,\max}$, $i \ge 0$, $r = 1, 2, \ldots, m$ and $\left|x_r(k+i|k)\right| \le x_{r,\max}$, $i \ge 1$, $r = 1, 2, \ldots, n$. Let $x(k|k)$ be the measured state $x(k)$ at sample time k that satisfies the constraints. Then, the state feedback matrix $F(k)$ in the control law $u(k+i|k) = F(k)x(k+i|k)$ that minimizes the upper bound $V(x(k|k))$ of the objective function $J(k)$ at time instant k is given by $F = YG^{-1}$, where G and Y are obtained from the solution of the following optimization problem with variables M, G, Y, X, W, ξ, γ, and $Z = [G; Y]$ and ε is a positive coefficient.*

$$\min_{M,G,Y,Z,X,W,\xi,\gamma} \gamma \quad (8)$$

subject to

$$\begin{bmatrix} I & x(k|k)^T \\ x(k|k) & M \end{bmatrix} \ge 0, \quad (9)$$

$$\begin{bmatrix} G + G^T - M & * & * & * & * \\ \sqrt{(1+\varepsilon)}(AG + BY) & M & * & * & * \\ \sqrt{(1+\varepsilon^{-1})}LZ & 0 & \xi I & * & * \\ Q^{1/2}G & 0 & 0 & \gamma I & * \\ R^{1/2}Y & 0 & 0 & 0 & \gamma I \end{bmatrix} \ge 0, \quad (10)$$

$$M - \xi I > 0, \quad (11)$$

$$X - M \ge 0 \quad \text{with} \quad X_{rr} \le x_{r,\max}^2, \quad r = 1, 2, \ldots, n. \quad (12)$$

$$\begin{bmatrix} W & Y \\ Y^T & G + G^T - M \end{bmatrix} \ge 0 \quad \text{with}\, W_{rr} \le u_{r,\max}^2, \, r = 1, 2, \ldots, m. \quad (13)$$

Proof. Consider the following lemma to develop the proof.

Lemma 1. *Let $\bar{M}$ and $\bar{N}$ be real constant matrices and P be a positive matrix of compatible dimensions. Then $\bar{M}^T P \bar{N} + \bar{N}^T P \bar{M} \le \varepsilon \bar{M}^T P \bar{M} + \varepsilon^{-1} \bar{N}^T P \bar{N}$ holds for any $\varepsilon > 0$.*

Proof. See [9] for the proof and finding the optimal value for ε.□

By applying Schur complements to $V(x(k|k)) \le \gamma$ in (7) and substituting $P = \gamma M^{-1}$, (9) is derived. To obtain (10), (5) can be rewritten as

$$x(k+i+1|k)^T P x(k+i+1|k) - x(k+i|k)^T P x(k+i|k) + x(k+i|k)^T Q x(k+i|k) + u(k+i|k)^T R u(k+i|k) < 0. \quad (14)$$

Based on Lemma 1, the following inequality is derived for the first term in (14).

$$\begin{aligned} & x(k+i+1|k)^T P x(k+i+1|k) \\ & = (Ax(k+i|k) + Bu(k+i|k) + \tilde{f}(x(k+i|k), u(k+i)))^T P(Ax(k+i|k) \\ & + Bu(k+i|k) + \tilde{f}(x(k+i|k), u(k+i|k))) \\ & \le (1+\varepsilon)(Ax(k+i|k) + Bu(k+i|k))^T P(Ax(k+i|k) + Bu(k+i|k)) \\ & + (1+\varepsilon^{-1})\tilde{f}(x(k+i|k), u(k+i|k))^T P \tilde{f}(x(k+i), u(k+i|k)). \end{aligned} \quad (15)$$

By considering $P < \mu I$ (μ is the largest eigen value of P) and using (3) and (15), (14) can be expressed as