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Minimal required excitation for closed-loop identification: Some implications for data-driven, system identification ☆

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ABSTRACT

The development and effective use of all available data is extremely important. Previous work has shown that it is possible to identify process models using closed-loop data even if the reference signal was not being excited. However, such results require that the system have a sufficiently large time delay or alternatively a fast sampling time. Therefore, this paper seeks to examine and provide general results for identifiability of a process using closed-loop data with or without changes in the reference signal. Similarly to the previous case, it is shown that the complexity of the required reference signal depends strongly on the sampling time and time delay. However, since many fast processes without time delay can be modelled as first-order systems, they can indeed be identified when the excitation in the reference signal is a simple step function or sequence of such functions. Using numerical simulations as well as the Tennessee Eastman process, the effect on the continuous time parameters is investigated for different sampling times and excitations signals. It is shown that as expected an external reference signal can identify previously difficult-to-identifiable cases.

1. Introduction

Industrially, the development and assessment of models form an important aspect of many different control strategies, including fault detection and diagnosis, advanced process control, and process optimisation. In many cases, it is desired to keep plant perturbations, that is, changes, as small as possible. Although it would be convenient if all models could be determined without plant perturbations, it has been shown that in systems with small time delays or very large sampling times, this may not be feasible [1]. It is well known that white noise or other sufficiently complex perturbations can excite any system. However, such random or large excitations can cause unnecessary process variability or angst amongst the operators. For this reason, it would be useful to determine relevant conditions that allow the determination of the minimal signal properties required for the reference signal in order to identify a model of the system.

The foundations of determining the conditions for identifying discrete models from closed-loop data started with the work of Box, McGregor, Söderström, and Stoica, who sought to determine the theoretical limits on the delay to guarantee consistency of parameter estimates in the absence of a reference signal [2–4]. This research led to the development of general conditions for closed-loop identification. In order to obtain a solution various assumptions were made including dealing with an autoregressive moving average model with exogenous input (ARMAX) with at least a single sample delay [4] or various degrees of *a priori* knowledge of pole-zero cancellations in the closed-loop transfer function [5–7]. Removing these conditions would provide general results that would then have broad applicability. The general case for closed-loop identification with no perturbations in the reference signal has been recently completed [1]. However, there has not been any work done to extend these results to the case where there are perturbations in the reference signal. Preliminary results were presented in [8].

Therefore, the objectives of this paper are (1) to extend the previously developed closed-loop identification results to the case where there are changes in the references signal for all types of controls and discrete, linear models; (2) using the resulting conditions to examine relationships between model orders, reference signal excitation, and sampling time; and (3) finally, to examine the results using both simulations and experimental data.

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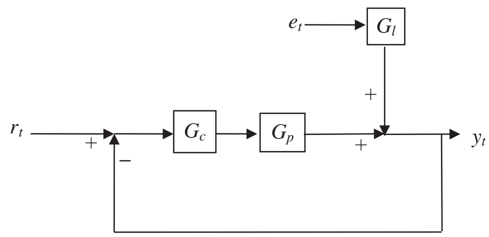

Fig. 1. Generic closed-loop process.

2. Identifiability in closed-loop systems

2.1. Theoretical results

Consider a process that can be modelled as a closed-loop prediction error (PE) system, similar to that shown in Fig. 1, i.e.,

$$G_c = \frac{X(z^{-1})}{Y(z^{-1})}y_t, \quad G_p = \frac{z^{-n_k}B(z^{-1})}{A(z^{-1})F(z^{-1})}, \quad G_l = \frac{C(z^{-1})}{A(z^{-1})D(z^{-1})} \tag{1}$$

where the X-polynomial is given as

$$X(z^{-1}) = 1 + \sum_{i=1}^{n_X} x_i z^{-i} \tag{2}$$

n_X is the order of the polynomial; the Y-, A-, C-, D-, and F-polynomials are defined similarly to the X-polynomial; the B-polynomial is defined as

$$B(z^{-1}) = \sum_{i=1}^{n_B} \beta_i z^{-i} \tag{3}$$

n_B is the order of the B-polynomial; and n_k is the time delay in the process, which excludes the one sample time delay introduced by the sampler. For ease of presentation, the backshift operator, z^{-1}, will be dropped in the following sections, unless it is required for clarity or emphasis.

For this system, the one-step ahead predictor, $y(t|t–1, \theta)$, is

$$y(t|t-1,\theta) = \underbrace{G_l^{-1}G_p}_{W_u}u_t + \underbrace{(1-G_l^{-1})}_{W_y}y_t \tag{4}$$

Let a quasistationary vector signal, ψ_t, be persistently exciting if $\bar{E}(\psi_t\psi_t^T) > 0$ [9]. A quasistationary vector signal, r_t, is said to be sufficiently rich of order n_r if the following regressor is persistently exciting,

$$\varphi \equiv \begin{bmatrix} r(t-1) \\ r(t-2) \\ \vdots \\ r(t-n) \end{bmatrix} = \begin{bmatrix} z^{-1} \\ z^{-2} \\ \vdots \\ z^{-n} \end{bmatrix} r_t \tag{5}$$

In order to distinguish between any two candidate models for a given closed-loop data set, it is necessary that the following two conditions hold [9]:

$$\begin{cases} E((\Delta W_y r_t)^2) = 0 \\ \Delta W_y \equiv G_c \Delta W_u \end{cases} \tag{6}$$

where

$$W_u = G_l^{-1}G_p = \frac{z^{-n_k}DB}{CF}, \quad W_y = 1 - G_l^{-1} = \frac{C-AD}{C} \tag{7}$$

and Δ represents the difference between the two candidate models 1 and 2, i.e.,

$$\Delta W = W_1 - W_2 \tag{8}$$

Substituting the results from Eq. (7) into Eq. (6) gives

$$\begin{cases} E\left(\left[\left(\dfrac{A_2D_2}{C_2} - \dfrac{A_1D_1}{C_1}\right) r_t\right]^2\right) = 0 \\ \left(\dfrac{X}{Y}\right)\left(\dfrac{z^{-n_k}D_1B_1}{C_1F_1} - \dfrac{z^{-n_k}D_2B_2}{C_2F_2}\right) = \left(\dfrac{A_2D_2}{C_2} - \dfrac{A_1D_1}{C_1}\right) \end{cases} \tag{9}$$

In order to solve for the closed-loop conditions, consider that

(1) There are possible cancellations between D_1 and F_1, so that $D_1 = H\bar{D}_1$ and $F_1 = H\bar{F}_1$, where H is a polynomial with order n_H and $\bar{D}_1$ and $\bar{F}_1$ are coprime.
(2) There are possible cancellations between $C_1\bar{F}_1Y$ and $(\bar{F}_1A_1HY + z^{-n_k}B_1X)$, so that

$$\begin{aligned} &C_1\bar{F}_1Y = TC_1\bar{\bar{F}}_1Y = T\bar{C}_1\bar{\bar{F}}_1\bar{Y} \\ &(\bar{F}_1A_1HY + z^{-n_k}B_1X) = T(\overline{\bar{F}_1A_1HY + z^{-n_k}B_1X}) \end{aligned} \tag{10}$$

where T is a polynomial with order $n_T = \min(n_C + n_F + n_Y, n_F + n_A + n_Y, n_B + n_X)$ and $T\bar{C}_1\bar{\bar{F}}_1\bar{Y}$ and $(\overline{\bar{F}_1A_1HY + z^{-n_k}B_1X})$ are coprime. This takes into consideration any potential pole-zero cancellations in the closed-loop system transfer function. It should be noted that since the common terms in the denominator may only appear after the terms in the denominator have been combined, it can be shown that [1]

$$\bar{F}_1A_1HY + z^{-n_k}B_1X = M_1N + P_1O = T\bar{M}_1\bar{N} + T\bar{P}_1\bar{O} \tag{11}$$

where M_1, N, P_1, and O are polynomials constructed so that the common terms between numerator and denominator of the closed-loop transfer function given by the T-polynomial, appear in both terms of the sum, the orders of the M_1 and P_1 are, respectively, equal to that of the sum of the A_1- and F_1-polynomials and the B_1-polynomials, and, finally, $\bar{N}$ and $\bar{O}$ are coprime. The number of overbars placed over the polynomials represents the number of potential reductions in the order of the polynomial due to non-coprimedness of the selected polynomials.

Theorem 1 (Routine-operating case). *Assume that the reference signal remains constant with a value of zero and the assumptions described above hold, then the system can be identified if the following relationship holds among the orders of the polynomials and the discrete time delay*:

$$\max\begin{pmatrix} n_X + n_k - n_F - n_A, \\ n_Y - n_B \end{pmatrix} \geq n_D + \min\begin{pmatrix} n_C + n_F + n_Y, \\ n_A + n_F + n_Y, \\ n_B + n_X \end{pmatrix} \tag{12}$$

Proof. Since the proof follows the same approach as used in [10] and is fully proven in [11], it is omitted here.□

Theorem 2 (Excited reference signal case). *Assume that the assumptions described above hold, then the minimal excitation required for the reference signal is*

$$\begin{aligned} n_r \geq\; & n_D + \min(n_C + n_F + n_Y, n_A + n_F + n_Y, n_B + n_X) \\ & + \min(n_F + n_A - n_X - n_k, n_B - n_Y) \end{aligned} \tag{13}$$

with the caveat being that the points of support for r_t do not coincide with any possible zeroes of X on the unit circle. The richness order of the reference signal is denoted by n_r and is defined by Eq. (5).

Proof. The proof of this theorem is given in Appendix I.□