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An extended Linear Quadratic Regulator with zone control and input targets

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ABSTRACT

This paper focuses on the application of the Linear Quadratic Regulator (LQR) in industrial process systems where one has zone control of the outputs and input economic targets as well as constraints on the inputs and input moves. In the approach proposed here, the LQR is combined with the Model Predictive Control (MPC) in a framework where the system outputs are controlled through the LQR state feedback control law and the output set points are manipulated by the MPC to enforce the input constraints and input targets. The performance of the proposed controller is tested through the simulation of the control of an industrial deisobutanizer column.

1. Introduction

In the process industry, the advanced control problem of constrained multivariable systems is usually tackled with model predictive control [1]. Based on a model of the system to be controlled, MPC calculates at each time step an open-loop finite sequence of manipulated inputs that optimizes the predicted behavior of the system, usually subject to constraints on the inputs and outputs. When the closed-loop system is considered, the open-loop strategy of MPC shows to be suboptimal because it is based on finite input and output prediction horizons. Also, the stability of the closed-loop system depends on the tuning parameters even when the process model is perfectly known.

In the unconstrained case, MPC with infinite input and output horizons becomes equivalent to the Linear Quadratic Regulator [2] where a state feedback control law can be expressed as $u(k+i)=K(k)x(k+i)$, $i \geq 0$. Besides being optimal in closed-loop, another advantage of LQR is that stability can be assured in the state feedback case and the separation principle allows the inclusion of a state estimator such that stability is extended to the output feedback case. In the set point tracking case, the constrained LQR can be implemented through iterative procedures where MPC and LQR are combined to compute the input sequence along the infinite horizon [3,4]. Kothare et al. [5] proposed a strategy that is an extension of the LQR approach to the constrained case. Conservative LMI constraints on the inputs and outputs are added to the optimization problem that defines the controller so that the solution to the constrained case becomes a state feedback control law as in the unconstrained case. For the practical application, the approach suffers from some limitations as the conservative way the constraints are implemented reduces the attraction domain of the controller, the zone control strategy [6] is not addressed and the large number of decision variables impacts the computational burden of the control problem. In the zone control strategy, the output does not have a fixed set-point and it has only to be controlled inside a zone or range.

For the finite horizon MPC, stability can be achieved through the inclusion of a state constraint at a finite time step [7–11]. The main drawback of this approach is that when, because of the input constraints, the state constraint cannot be satisfied, the problem that defines the MPC becomes infeasible, which is not acceptable in practice. Stability of the MPC can also be achieved through the extension of the output prediction horizon to infinity [12,13]. A limitation of this method to the case of unstable systems is the need to include additional constraints to zero the effect of the unstable modes beyond the control horizon. These additional constraints may also conflict with the input constraints and turn the MPC infeasible.

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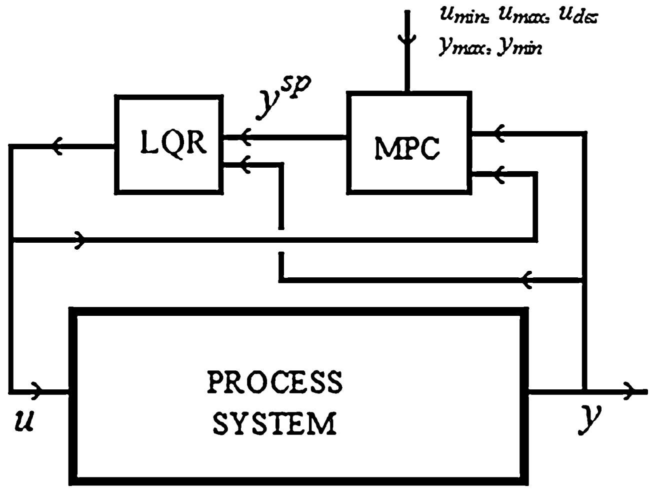

Fig. 1. Schematic representation of the combined LQR–MPC approach.

To avoid the use of a state observer, one can use a state space model as the realigned state model in which the state is composed of the past measured outputs and inputs of the system [6]. Recent papers dealing with model predictive control based on such non-minimal model includes Wang and Young [14], González et al. [15] and Zhang et al. [16]. In the simulation example considered here, the model representation proposed by González et al. [15] is adopted.

In the process industry, MPC is usually implemented in a multi-layer structure. The usual hierarchical control architectures based on MPC are summarized by Scattolini [17]. More recently, simplified versions of the hierarchical structure, which integrate the multi-layer structure into a single layer have been proposed [18–20], but the simplified approach has not been extensively implemented in industry yet.

In this paper, as illustrated in Fig. 1, focusing on the implementation on real systems of the process industry, a new approach where the state feedback LQR control strategy is combined with the MPC framework in order to address the zone control strategy of process systems where an upper layer in the control structure defines economic targets for some of the inputs is proposed. The objective is then to drive the system to these targets while maintaining the output inside predefined boundaries. In the approach proposed in this work, the process outputs are controlled through the LQR control law, but the output set-points are not fixed and are allowed to vary inside the output control zones. Then, in the proposed structure, the output set points become the actual manipulated variables of the MPC. This strategy allows the constraints on the process inputs to be addressed at the MPC level through the computation of suitable output set points. Finally, since the closed-loop resulting from the application of the LQR feedback control law to the process system is stable even if the open loop process system is unstable, the proof of convergence and stability of the LQR–MPC approach is simplified.

This paper is organized as follows. In Section 2, the application of the state feedback control law to the process model is discussed and the resulting state space representation of the closed loop with the LQR is developed. In Section 3, the MPC that manipulates the output set points and considers zone control of the process outputs and optimizing targets for the process inputs is formulated. The stability and convergence of the proposed approach are discussed. In Section 4, the performance of the proposed control approach is illustrated through the simulation of the control of an industrial deisobutanizer column. Finally, in Section 5 the paper is concluded.

2. The process model for the LQR–MPC

Assume that one has a controllable system that can be represented by a state space model in the incremental form as by González et al. [15]:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where, $x \in \Re^{nx}$, $u \in \Re^{nu}$, $\Delta u(k) = u(k) - u(k-1)$, $y \in \Re^{ny}$ and A, B and C are matrices of appropriate dimensions. The system represented in Eq. (1) has poles at +1 because of the input incremental form of the model and it is assumed that the system may have other stable or unstable poles.

Now, suppose that one stabilizes the system represented in Eq. (1) through the following state feedback control law:

$$\Delta u(k) = F\left[x(k) - x^{sp}(k)\right] \tag{2}$$

where, $x^{sp}(k)$ is the state set-point at time step k and assume that $x^{sp}(k) = \tilde{I} y^{sp}(k)$ where $\tilde{I}$ is known and $y^{sp}(k)$ is the output set-point at time step k.

F is the optimal gain matrix such that in closed loop, the state feedback control law defined in Eq. (2) minimizes the cost function:

$$J_{\infty,k} = \sum_{j=0}^{\infty} \left[y(k+j|k) - y^{sp}\right]^T Q_y \left[y(k+j|k) - y^{sp}\right] + \sum_{j=0}^{\infty} \Delta u(k+j|k)^T R_{\Delta u} \Delta u(k+j|k) \tag{3}$$

where, Q_y and $R_{\Delta u}$ are positive definite weight matrices, which are assumed to be known before the computation of F.