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The design of a non-minimal state space fractional-order predictive functional controller for fractional systems of arbitrary order

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ABSTRACT

In this paper, the design of the fractional-order predictive functional controller ($^{\alpha}$PFC) for the linear fractional systems of arbitrary order has been presented. For this purpose, at first, the fractional order transfer function has been digitized via Grunwald–Letnikov definition to obtain the linear regression model of the system. Next, the non-minimal input–output fractional-order state space ($^{\alpha}$NMSS) model of the system has been derived. The fractional-order predictive functional controller ($^{\alpha}$PFC) has been then designed for the $^{\alpha}$NMSS model structure via defining a fractional order cost function over the fractional-order non-minimal state vector. Finally, genetic algorithm (GA) has been employed to obtain the optimal $^{\alpha}$PFC control coefficients. The fractional-order model of two rods thermal bench has been considered as the uncertain case study in this paper. Simulation results for temperature control of this fractional-order system are representative of better performance of the designed controller with respect to the NMSSPFC as well as the fractional GPC.

1. Introduction

Model predictive control (MPC) and its variants have experienced intensive study in control theory and engineering since the 1970s. Typically, there are three main MPC design methods: finite impulse response (FIR) or step response models, transfer function models and state space models [1]. There are several state-space MPC variants in the literature [1–10]. Amongst, predictive functional control (PFC) is the most popular one which has been employed widely in both researches and industrial applications [11,12]. PFC has been pioneered by Richalet for control of dynamically fast systems [12,13]. It uses concepts of prediction and control horizons as other MPC variants. But, due to its intrinsic ability to handle input and state constraints for large scale multivariable plants as well as based on its simple formulation and ease of understanding, PFC has been in use in the process industries, where it has become an industrial MPC standard. For example, in [14], an ARM–PFC controller has been developed for active queue management in congested TCP/IP networks. In [15], design and implementation of a speed controller based on predictive functional control for the PMSM system has been investigated. In [16], it has been used for solving the problem of conveyor belt speed control in a factory for producing stone wool. In [17], predictive functional control technique has been applied for temperature control of a bench-scale batch reactor and implemented via Programmable Logic Controller (PLC). In [18], a partially decoupled design of the state space PFC has been presented for MIMO processes. The multivariable process has been first treated into MISO process by a simple Cramer's rule solution to linear equations which has been provided a balance between model complexity and control system design, and then the MISO process based extended state space predictive functional control has been designed. In [19], an adaptive fuzzy PFC has been designed for control of a batch reactor. This system is a nonlinear time-varying process which its dynamic has been encountered via a fuzzy identification approach and is controlled by a fuzzy PFC. In [20], the design and robustness analysis of PFC for unstable SISO systems with arbitrary number of unstable poles has been considered which is based on coprime-factorized process model. In [21] and [22], the MPC control of hybrid system with both discrete and continuous inputs has been considered. Especially, in [21], the control of the temperature in an exothermic batch reactor via self-adaptive PFC has been dealt with. This system is a hybrid system with both discrete and continuous inputs. The simulation results of [21] are representative of the good ability of PFC in control of such a complicated system.

The design of hybrid MPC as well as hybrid PFC controllers has been considered in some researches, in which, search algorithms such genetic algorithm (GA) has been used for solving the

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optimization problems. For example, in [23], a hybrid fuzzy predictive control based on GA has been designed for the temperature control of a batch reactor. In [24], PFC based PID design using GA has been developed for control of chamber pressure in a cook furnace. In [25], GA-based PFC for control of batch processes under actuator faults has been considered.

One of the major drawbacks which exist in the design of all state space MPC variants such as PFC is the problems originated from the use of observers in such control systems. That is, the observer dynamics should be faster than that of the state feedback controller which in turn leads to numerical difficulties. Besides, in case of activating the process constraints, the closed loop performance will significantly deteriorated as a kind of nonlinearity dominating the control system [1]. In order to tackle the above problems, in [26], a non-minimal state space (NMSS) model has been proposed in which the measured process inputs, outputs and their past measured values are incorporated to a non-minimal state vector employed in MPC design. Next, in [1,27–31], this idea has been extended and a new NMSS model structure is developed in which the output tracking error has been also incorporated. Based on this new NMSS model PFC has been formulated for various kinds of processes. For example in [27], the design on NMSSPFC for inverse-response processes and in [28], the design on NMSSPFC for MIMO processes, have been proposed. Successful application of the proposed method for several industrial case studies such as industrial coke furnace [29], injection molding batch process [30] and a coke fractionation tower [31] are representative of good flexibility and capability of the proposed infrastructure in dealing with various processes.

One of the most important factors on the effectiveness of the process control systems and specially model based controllers is the quality and precision of the model [32,33]. One of the more effective methods employed to describe real properties exhibited by process industries is the technique based on fractional-order derivatives [32,34]. It is due to the extra degrees of freedom and the flexibility which can be used to precisely fit the experimental data much better than the integer-order modeling. Therefore, the fractional calculus has become a powerful tool in describing the dynamics of complex systems which appear frequently in several branches of science and engineering and found numerous applications in the field of viscoelasticity, robotics, feedback amplifiers, electrical circuits, control theory, electro analytical chemistry, fractional multi-poles, chemistry and biological sciences [4,35–40].

From another point of view, in the recent years, the fractional order controllers have been widely used to enhance the performance of the control systems [32]. That is they have the ability to improve the control performance with respect to the integer ones [32–42]. Therefore, various types of fractional-order controllers have been proposed in the literature. In [43], the standard H_∞ control problem for continuous-time fractional linear time-invariant SISO systems has been stated. In [44], several decomposed hybrid structures of the fractional order fuzzy PID controller has been proposed. In [45], classical proper PID controllers have been designed for linear time invariant fractional-order systems with time delays. In [46], a LMI-based design method of robust control has been presented for fractional-order uncertain systems with the fractional order $0 < \alpha < 2$; where, the sufficient conditions for the robust asymptotical stability of the fractional-order closed loop control systems was first presented. In [47], fractional order sliding mode controller has been suggested for antilock braking systems to regulate the slip to the desired value.

Fractional order predictive control has been in use in control of processes from 2008. In [48], the use of Generalized Predictive Control (GPC) with fractional order plants has been proposed. A low integer-order discrete approximation has been used as models to design the controllers. In [49,50] a generalization of GPC has been presented in which fractional-order operators are employed in its cost function. In [51] based on the fractional-order PID control algorithm and dynamic matrix control (DMC) algorithm, the fractional order PID dynamic matrix control (FOPID-DMC) algorithm has been introduced. In [52], a fractional-order GPC has been applied for low-speed control gasoline-propelled cars. A good survey about the design and usage of fractional-order GPC has been provided in [53]. In most of the developed fractional-order GPC in the literature, a transfer function model of rational order and a fractional-order cost function has been assumed. In [54], fuzzy control of fractional-order nonlinear discrete time processes has been dealt with. In [55], GPC has been applied to control of fractional thermal system. In designing the controller, the plant has been approximated with an integer one via Oustaloup's method [42]. In [32] switched state MPC has been designed for the fractional-order nonlinear discrete-time systems. In [36], a fractional-order ARX model based on orthonormal basis filter parameterization has been developed to be identified for formulating MPC.

As seen in the above, the design of fractional PFC has not been considered, in the literature. In this paper, the design of the fractional order predictive functional controller (${}^{\alpha}$PFC) for the linear fractional systems of arbitrary order has been considered. For this purpose, at first, the fractional order transfer function has been digitized via well-known Grunwald–Letnikov fractional derivative definition to obtain the linear regression model of system. Next, the non-minimal input–output fractional-order state space (${}^{\alpha}$NMSS) model of the system has been derived. The fractional-order predictive functional controller (${}^{\alpha}$PFC) has been then designed for the ${}^{\alpha}$NMSS model structure via defining a fractional order cost function over the fractional non-minimal state vector. Finally, genetic algorithm (GA) has been employed to obtain the optimal ${}^{\alpha}$PFC control coefficients. The fractional-order model of two rods thermal bench [56] has been considered as our uncertain case study in this paper. A thorough study about the performance of the proposed controller has been done such as comparison with some other controllers, sensitivity and robustness analyses as well as investigation of its performance under input constraints. Simulation results for control of this fractional-order system are representative of better performance of the designed controller with respect to NMSSPFC as well as fractional GPC.

The rest of this paper has been organized as follows. In Section 2, the preliminary materials including the basics of fractional order calculus, the fractional-order models and GA algorithm is described. The formulation of ${}^{\alpha}$NMSS model is presented in Section 3 and in Section 4 the design of ${}^{\alpha}$PFC for the fractional-order systems has been described. The simulation results of ${}^{\alpha}$PFC for the fractional thermal system is represented in Section 5. Finally, conclusions of this paper will be expressed in Section 6.

2. The preliminary materials

2.1. The basics of fractional order calculus

The fractional calculus is a field of mathematics study that grows out of the traditional definitions of the calculus integral and derivative operators in much the same way fractional exponents is an outgrowth of exponents with integer value [57]. Fractional calculus has its origin in the question of the extension of the meaning. A well known example is the extension of meaning of real numbers to complex numbers, and another is the extension of meaning of factorials of integers to factorials of complex numbers.

In generalized integration and differentiation, the question of the extension of the meaning is [42]: Can the meaning of derivatives of integral order $d^n y/dx^n$ be extended to have meaning where n is any number i.e. irrational, fractional or complex?