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Influence of higher-order effects on pulsating solutions, stationary solutions and moving fronts in the presence of linear and nonlinear gain/loss and spectral filtering

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ABSTRACT

We have studied the impact of the higher-order effects: intrapulse Raman scattering (IRS), third-order of dispersion (TOD) and self-steepening (SS) on pulsating solutions, moving fronts and stationary solutions of the complex cubic–quintic Ginzburg–Landau equation (CCQGLE) found in Tsoy and Akhmediev (2005) as well as on the solutions presented in Uzunov et al. (2014). The applied basic equation generalizes the CCQGLE with the IRS, TOD and SS effects. A finite-dimensional dynamical system has been derived using the method of moments. Applying the derived dynamical system alongside with the numerical solution of the generalized CCQGLE performed by means of the fourth-order Runge–Kutta interaction picture method we have found that the influence of IRS and SS is stronger than the impact of TOD for the solutions of Tsoy and Akhmediev (2005). Perturbed pulsating solutions, moving fronts and stationary solutions in the presence of IRS, SS and TOD have been numerically observed. They exist up to some critical values of the parameters of perturbations. For the values of parameters larger than the critical ones the pulsating solutions are transformed into stable stationary solutions or unstable solutions. New localized fluctuating and stationary solutions have been obtained for fairly large values of parameters of IRS and TOD, respectively. The transformation of the stable stationary solution of Uzunov et al. (2014) under the influence of SS into pulsating solution has been numerically observed.

1. Introduction

It is well known that the complex cubic–quintic Ginzburg–Landau equation (CCQGLE) in optics [1–4] can model soliton transmission lines [5,6] as well as passively mode-locked laser systems [7–10].

The exact solutions of the complex cubic Ginzburg–Landau equation (CCGLE) has been reported in [11–14]. An exact solution of the CCQGLE has been proposed in [6]. The numerical solutions of CCQGLE could be divided into two groups – localized fixed-shape solutions and localized pulsating solutions. Localized fixed-shape solutions are the stable stationary pulses, the composite pulses and the moving pulses [1]. Localized pulsating solutions can be plain pulsating, creeping, snaking and erupting solutions [15,16]. Chaotic pulsating and period doubling solutions were observed in [17–18].

For the analysis of CCQGLE there has further been proposed a reduction to a finite dimensional system by means of the method of moments [19,20]. The bifurcation analysis of optical solitons in CCQGLE has been reported in [19,20]. It has been shown that the pulsating solutions correspond to the limit cycles of the finite dimensional dynamical system. It has been shown that the periodic, quasi-periodic and chaotic attractors of Euler–Lagrange equations obtained through the Lagrangian method can be related to numerically observed solutions of CCQGLE [21,22].

It has been shown that narrowband filtering and nonlinear gain can be used to control the self-frequency shift due to the intrapulse Raman scattering (IRS) of ultra-short optical solitons in fiber-optic systems [23]. The role of the nonlinear gain is to give an effective gain to the soliton and suppression of the noise or, in other words, to reduce the background instability [23]. Recently, while analyzing the same physical situation, we numerically observed that the small change of the parameter describing IRS leads to qualitatively different behavior of the evolution of pulse amplitudes [24]. Using the first two moments of the nonlinear Schrödinger equation we have proved that the strong dependence of the pulse dynamics on the IRS is related to the existence of the Poincare–Andronov–

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Hopf bifurcation (PAHB) and the appearance of the unstable limit cycle [24].

The influence of the higher-order effects – the third-order of dispersion (TOD), IRS and the self-steepening (SS) on the fiber laser operation or on the stable stationary solutions of the CCGLE has been studied in [25]. The existence of the exact chirped solitary solution of this generalized CCGLE, which includes the higher order effects, has been first reported in [26]. Recently it has been shown numerically that under the influence of the higher-order effects, the localized pulsating solutions of CCQGLE can be transformed into fixed-shape solutions for a certain range of parameter values [27,28].

An approach has been presented for the identification of periodic attractors of the generalized CCQGLE with IRS [29,30]. Using the ansatz of the traveling wave and fixing some relations between the material parameters, the strongly nonlinear Lienard–Van der Pol equation for the amplitude of the nonlinear wave has been derived [29,30]. Next, the Melnikov method has been applied to this equation and the existence of a limit cycle has been shown. The stability of this limit cycle has been proved in [29,30]. A perturbation theory for the description of the influence of the nonlinear refraction index, IRS and TOD on the stable stationary solutions below the PAHB identified in [24] has been proposed in [31]. This theory completes the theory used in [23,24] including in its analysis the position and phase of the soliton pulse. It turned out that the equations for the position and phase are independent from those of the amplitude and frequency. At the same time the higher-order effects influence only the equations of position and phase. So the model was not able to describe the pulsating solutions in the presence of higher-order effects. However, it has been predicted by this model and the numerically verified significant reduction of the time shift of stable stationary solutions due to the IRS in the presence of SS and TOD[31].

We study here the influence of the higher-order effects on moving fronts, pulsating and stationary solutions described in [6,19,20] as well as on the stationary and pulsating solutions found in [24]. One of our aims is to propose a general approach for the study of the influence of the IRS, SS and TOD on all of the solutions of CCQGLE avoiding the restrictions of previous approaches [29,30,24,31]. As it is well known the method of moments [32] could be useful for the analysis of stationary solutions, pulsating solutions and moving fronts [19,20]. In addition to the usual soliton perturbation theory we can also consider the evolution of the width that is not related to the amplitude and chirp. The last situation is typical for the solutions of CCQGLE [6]. We apply here the method of moments [32] in order to derive five dimensional dynamical systems of ODE for the evolution of the amplitude, frequency, position, width and chirp of all solutions of CCQGLE. The derived here dynamical system (see Eq. (3) below) generalizes the dynamical systems used earlier in [32,19,20]. For the case of small values of parameters, predictions of system (3) are verified by the numerical solution of the generalized CCQGLE performed by means of the fourth-order Runge–Kutta interaction picture method. In the case of large values of the parameters pure numerical investigation has been performed.

The paper is organized as follows: First, the physical meaning of the generalized CCQGLE is presented in Section 2. Section 3 contains the details of numerical simulation of Eq. (1). The derived dynamical system of ODE is introduced in Section 4. Section 5 gives the results of the analysis of the influence of higher-order effects on the solutions of CCQGLE [6,19,20]. In Section 6 we present the results of the analysis of the impact of higher-order effects on the stationary solution and pulsating solution of [24]. In Section 7 we study the performance of the derived System (3). Finally, we make our conclusions in Section 8. The appendix contains a short description of the application of the method of moments [32].

2. Basic equation

The propagation of ultra-short pulses in the presence of spectral filtering, linear and nonlinear gain/loss, as well as higher order effects: IRS, TOD and SS is described by the following generalized CCQGLE [4,27,28]:

$$i\frac{\partial U}{\partial x}+\frac{1}{2}\frac{\partial^2 U}{\partial t^2}+|U|^2U = i\delta U + i\beta\frac{\partial^2 U}{\partial t^2}+i\beta_3\frac{\partial^3 U}{\partial t^3}+i\varepsilon|U|^2U-\nu|U|^4U + i\mu|U|^4U - is\frac{\partial}{\partial t}(|U|^2U)+\gamma U\frac{\partial}{\partial t}(|U|^2) \quad (1)$$

where U is the normalized envelope of the electric field, X is the normalized propagation distance, t is the retarded time, δ is the linear gain or loss coefficient, β describes spectral filtering (gain dispersion), β_3 accounts for the TOD, ε is related to the nonlinear gain-absorption process, μ, if negative, accounts for the saturation of the nonlinear gain, ν, if negative, corresponds to the saturation of the nonlinear refraction index, s describes the self-steepening effect. The last term in Eq. (1) describes the IRS and γ is related to the first moment of the nonlinear response function (the slope of the Raman gain spectrum) [3,4].

3. Numerical solution of the basic equation

The numerical solution of Eq. (1) in this paper is performed by means of the "fourth-order Runge–Kutta method in the interaction picture method" (RK4IP method) RK4IP can generally be interpreted as an exponential Runge–Kutta method applied to the solution of parabolic PDE [33–37]. The method has a 4th-order convergence, which makes it a good alternative to all known variants of the split step Fourier methods (SSFM). In some cases, the method shows a significantly better accuracy than the SSFM [34,35,37]. A comparison of the performances of RK4IP and SSFM has been made for a number of cases from [36,38] and a perfect match has been obtained. The numerical parameters applied for the presented here numerical results are: sampling rate: 8192; time step 0.04; propagation step: 0.001. The typical distance for the calculation of the propagation of pulsating and stationary solutions in this work is for $x \approx 2000$. In some cases larger distances of calculation are used. These cases will be mentioned later in the text. For the calculation of the evolution of amplitudes, frequencies and time shifts of solutions there has been used the numerical data product Origin 8.5.

As the aim of this work is the study of localized solutions, we use the analysis of [20 and references therein] to choose the proper values of the physical parameters. Due to the relation between the existence of localized solutions and the existence of continuous waves we look for a plane wave in the form of $U = u\exp(iKx - i\omega t)$, where u, K and ω are real parameters. Substituting this expression in Eq. (1) we found the following equations for u and K: $u[\varepsilon u^2+\mu u^4 + (\delta-\beta\omega^2)] = 0$ and $K = \frac{1}{2}(2su^2\omega - \omega^2 + 2\beta_3\omega^3 + 2u^2 + 2\nu u^4)$. The higher- order terms included in this work influence only the second equation, so the analysis for the amplitudes of the solutions of the first equation performed in [20] remains valid. We assume in what follows that the following conditions will be satisfied: $\delta,\mu < 0$, $\beta,\varepsilon > 0$ and $\varepsilon > 2\sqrt{\mu(\delta-\beta\omega^2)}$.

The initial condition with which Eq. (1) is numerically solved is: $U(x,t) = U_0 sech(U_0 t)\exp(-i\omega_0 t)$ where U_0 and ω_0 are initial pulse amplitude and frequency, respectively.

4. Dynamical system of ODE

To derive the dynamical system we use the trial function in the form [32,19,20]: