Journal of Process Control 24 (2014) 1179–1186

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

On the performance of economic model predictive control with self-tuning terminal cost☆

Matthias A. Müller[a,1], David Angeli[b,c,*], Frank Allgöwer[a,1]

[a] Institute for Systems Theory and Automatic Control, University of Stuttgart, 70550 Stuttgart, Germany
[b] Department of Electrical and Electronic Engineering, Imperial College, London, UK
[c] Dipartimento di Ingegneria dell'Informazione, University of Florence, Italy

ARTICLE INFO

Article history:
Received 11 February 2014
Received in revised form 14 May 2014
Accepted 15 May 2014
Available online 21 June 2014

Keywords:
Economic model predictive control
Self-tuning terminal cost
Generalized terminal constraint

ABSTRACT

In this paper, we analyze the closed-loop performance of a recently introduced economic model predictive control (MPC) scheme with self-tuning terminal cost. To this end, we propose to use a generalized terminal region constraint instead of a generalized terminal equality constraint within the repeatedly solved optimization problem, which allows us to obtain improved closed-loop asymptotic average performance bounds. In particular, these bounds can be obtained a priori. We discuss how the necessary parameters for the generalized terminal region setting can be calculated, and we illustrate our findings with two numerical examples.

1. Introduction

In recent years, a variant of model predictive control (MPC) termed *economic MPC* has received an increasing amount of attention. In contrast to standard tracking (or stabilizing) MPC, the primary control objective in economic MPC is not the stabilization of a given setpoint (or trajectory to be tracked), but rather the optimization of a given general performance criterion, possibly related the economics of the considered process. On a technical level, this means that the cost function in economic MPC needs not be positive definite with respect to some setpoint, as is typically assumed in standard tracking MPC. In the literature, various properties of economic MPC such as average performance and convergence of the resulting closed-loop system, optimal steady-state operation and fulfillment of average constraints were studied using different assumptions and/or additional (terminal) constraints (see, e.g., [2–7]). Furthermore, various applications of economic MPC have recently been reported, such as the ones in [8–11].

In this paper, we study an MPC framework using a generalized terminal constraint, meaning that the endpoint of the predicted state sequence has to be equal to some arbitrary steady-state (or contained in a terminal region around an arbitrary steady-state) and not to a specific one. Such a generalized terminal constraint setting has first been proposed in the context of tracking MPC [12,13], and recently also in economic MPC [14–16]. The main benefits compared to a setting with fixed terminal point or terminal region constraint are a possibly much larger region of attraction and a guarantee of recursive feasibility even in case that the cost function (and hence also the optimal steady-state) changes online. Furthermore, in the context of economic MPC, a priori knowledge of the optimal steady-state is not required in [15,16], which is needed when using a fixed terminal constraint.

On the other hand, a disadvantage of using a generalized terminal constraint in economic MPC is that closed-loop performance guarantees are not as easily obtained as in case of a fixed terminal constraint. In particular, in [15], for general initial conditions closed-loop performance bounds are only obtained under an additional controllability assumption and by overriding the MPC algorithm, i.e., if necessary, following the previously optimal solution. In [16], an economic MPC algorithm with self-tuning terminal cost was proposed, inspired by the one in [15] with fixed terminal weight. As was shown in [16], the benefits of a self-tuning terminal weight compared to a fixed one are (i) that the terminal weight can possibly be kept much smaller, which can be good both for

☆ Parts of this paper will bee presented at the American Control Conference, Portland, Oregon, 2014 (see [1]).

* Corresponding author at: Department of Electrical and Electronic Engineering, Imperial College, London, UK. Tel.: +44 2075946283.

E-mail addresses: matthias.mueller@ist.uni-stuttgart.de (M.A. Müller), d.angeli@imperial.ac.uk (D. Angeli), frank.allgower@ist.uni-stuttgart.de (F. Allgöwer).

[1] Supported by the German Research Foundation (DFG) within the Priority Programme 1305 "Control Theory of Digitally Networked Dynamical Systems" and within the Cluster of Excellence in Simulation Technology (EXC 310/1) at the University of Stuttgart.

http://dx.doi.org/10.1016/j.jprocont.2014.05.009
0959-1524/

numerical and (closed-loop) performance reasons, and (ii) the possibility to obtain closed-loop average performance bounds without further controllability assumptions and without possibly overriding the MPC algorithm as done in [15, Algorithm 3]. However, the resulting average performance bounds obtained in [16] are rather of conceptual nature, in the sense that they can only be verified a posteriori. Namely, the closed-loop system outperforms in average the cost of the best steady-state achievable from the ω-limit set of the resulting closed-loop trajectory (see Section 2.3 for further details). This cannot, in general, be determined a priori.

The contribution of this paper is to develop an economic MPC scheme with generalized terminal constraint and self-tuning terminal cost for which improved and a priori verifiable bounds on the closed-loop asymptotic average performance can be obtained. To this end, we modify the scheme proposed in [16] and replace the generalized terminal equality constraint by a generalized terminal region constraint; this idea has also been used in the context of tracking MPC (see, e.g., [12,13]). This allows us to show that the closed-loop average performance is at least as good as a value corresponding to a local minimum of the stage cost function restricted to the set of feasible steady-states. For linear systems with convex cost and constraints, this results in the average performance being at least as good as the optimal steady-state, which recovers results obtained for a fixed terminal constraint [2,3].

The remainder of this paper is organized as follows. In Section 2, we introduce the proposed economic MPC scheme with self-tuning terminal cost and generalized terminal region constraint; furthermore, we briefly review the results obtained in [16] and show that they carry over to the modified setting considered in this paper. The main results of this paper concerning improved and a priori verifiable bounds for the closed-loop average performance are then given in Section 3. In Section 4, we discuss how the necessary parameters for the generalized terminal region setting can be calculated. We illustrate our findings with two numerical examples in Section 5, before concluding the paper in Section 6. We close this section by noting that parts of the results presented in this paper have also appeared in the conference version [1]. The main novelties of this paper compared to [1] are (i) that the complete proof of our main result is included in Section 3, (ii) the design procedure of the terminal ingredients for nonlinear systems (Section 4.2) and (iii) the numerical examples in Section 5 illustrating our results.

1.1. Notation

Let $\mathbb{I}_{\geq 0}$ denote the set of nonnegative integers, and $\mathbb{I}_{[a,b]}$ the set of all integers in the interval $[a, b] \subseteq \mathbb{R}$. We define $B_\varepsilon(y)$ to be the ball of radius $\varepsilon > 0$ around the point $y \in \mathbb{R}^n$, i.e., $B_\varepsilon(y) := \{x \in \mathbb{R}^n : |x - y| \leq \varepsilon\}$. For a function $g : \mathbb{R}^n \to \mathbb{R}$, $g_x(y)$ denotes the gradient and $g_{xx}(y)$ the Hessian of g with respect to x, evaluated at the point $y \in \mathbb{R}^n$. Given two sets $\mathcal{A}, \mathcal{B} \subseteq \mathbb{R}^n$, the Minkowski set addition and Pontryagin set difference are defined as $\mathcal{A} \oplus \mathcal{B} := \{a + b : a \in \mathcal{A}, b \in \mathcal{B}\}$ and $\mathcal{A} \ominus \mathcal{B} := \{a \in \mathcal{A} : a + b \in \mathcal{A}\, \forall\, b \in \mathcal{B}\}$, respectively. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, denote by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ its minimum and maximum eigenvalue, respectively.

2. Economic MPC with self-tuning terminal cost

We consider discrete-time nonlinear systems of the form

$$x(t+1) = f(x(t), u(t)), \quad x(0) = x_0, \tag{1}$$

with $x(t) \in \mathbb{X} \subseteq \mathbb{R}^n$ and $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ for all $t \in \mathbb{I}_{\geq 0}$, and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is assumed to be continuous. The system is subject to (possibly coupled) pointwise-in-time state and input constraints $(x(t), u(t)) \in \mathbb{Z}$ for all $t \in \mathbb{I}_{\geq 0}$, where $\mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U}$ is assumed to be compact. Denote by $\mathbb{Z}_{\mathbb{X}}$ the projection of $\mathbb{Z}$ on $\mathbb{X}$. The control objective is to design a control law such that the resulting closed-loop system satisfies the given state and input constraints and such that a desired objective function ℓ is minimized. Here, the stage cost $\ell : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ is assumed to be continuous, but can otherwise be an arbitrary, possibly economic, function which need not satisfy any convexity or definiteness assumption. Furthermore, by (x_s, u_s) we denote an optimal steady-state achieving the minimal cost of all steady-states in the set $\mathbb{Z}$, i.e., (x_s, u_s) satisfies

$$\ell(x_s, u_s) = \min_{(x,u)\in\mathbb{Z},\, x=f(x,u)} \ell(x, u). \tag{2}$$

Note that as ℓ is continuous and $\mathbb{Z}$ is compact, we can assume without loss of generality that $\ell(x_s, u_s) = 0$.

2.1. Generalized terminal state constraint

For the setting as described above, in [16] we proposed the following economic MPC scheme with a self-tuning terminal cost, which we briefly recall in the following for the sake of completeness; this is a variation of the one introduced in [15] with fixed terminal weight. Namely, at each time t with $x := x(t)$, the following optimization problem is solved:

$$\min_{u(0|t),\ldots,u(N|t)} \sum_{k=0}^{N-1} \ell(x(k|t), u(k|t)) + \beta(t)\ell(x(N|t), u(N|t)) \tag{3}$$

subject to

$$x(0|t) = x \tag{4a}$$

$$x(k+1|t) = f(x(k|t), u(k|t)) \quad k \in \mathbb{I}_{[0,N-1]} \tag{4b}$$

$$(x(k|t), u(k|t)) \in \mathbb{Z}, \quad k \in \mathbb{I}_{[0,N]} \tag{4c}$$

$$x(N|t) = f(x(N|t), u(N|t)), \tag{4d}$$

$$\ell(x(N|t), u(N|t)) \leq \kappa(t), \tag{4e}$$

for some possibly time-varying terminal weight β and κ specified later. The notation $x(\cdot\,|t)$ and $u(\cdot\,|t)$ denote predicted state and input values (predicted at time t), respectively. As already discussed in the introduction, the main advantages of using a generalized terminal state constraint (4d) instead of a fixed terminal point constraint lie in the fact that a possibly much larger region of attraction is obtained, and that the optimal steady-state (x_s, u_s), which is normally used as a fixed terminal point constraint [2], does not have to be known a priori.

2.2. Generalized terminal region constraint

In this paper we propose a relaxed form of the MPC algorithm (3)–(4). Namely, instead of requiring the terminal predicted state to be equal to *some* steady-state as in (4d), we require the terminal predicted state to lie in a terminal region $\mathbb{X}^f(\bar{x})$ around *some* steady-state $\bar{x}$. This leads to the following optimization problem to be solved at each time instant t with $x := x(t)$:

$$\min_{u(0|t),\ldots,u(N-1|t),\bar{x}(t),\bar{u}(t)} \sum_{k=0}^{N-1} \ell(x(k|t), u(k|t)) + V_f(x(N|t), \bar{x}(t)) + \beta(t)\ell(\bar{x}(t), \bar{u}(t)) \tag{5}$$

subject to

$$x(0|t) = x \tag{6a}$$

$$x(k+1|t) = f(x(k|t), u(k|t)), \quad k \in \mathbb{I}_{[0,N-1]} \tag{6b}$$

$$(x(k|t), u(k|t)) \in \mathbb{Z}, \quad k \in \mathbb{I}_{[0,N-1]} \tag{6c}$$