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Spatial, temporal, and spatio-temporal modulational instabilities in a planar dual-core waveguide

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ABSTRACT

We investigate modulational instability (MI) in a planar dual-core waveguide (DWG), with a Kerr and non-Kerr polarizations based on coupled nonlinear Schrödinger equations in the presence of linear coupling term, coupling coefficient dispersion (CCD) and other higher order effects such as third order dispersion (TOD), fourth order dispersion (FOD), and self-steepening (ss). By employing a standard linear stability analysis, we obtain analytically, an explicit expression for the MI growth rate as a function of spatial and temporal frequencies of the perturbation and the material response time. Pertinently, we explicate three different types of MI—spatial, temporal, and spatio-temporal MI for symmetric/antisymmetric continuous wave (cw), and spatial MI for asymmetric cw, and emphasize that the earlier studies on MI in DWG do not account for this physics. Essentially, we discuss two cases: (i) the case for which the two waveguides are linearly coupled and the CCD term plays no role and (ii) the case for which the linear coupling term is zero and the CCD term is nonzero. In the former case, we find that the MI growth rate in the three different types of MI, seriously depends on the coupling term, quintic nonlinearity, FOD, and ss. In the later case, the presence of quintic nonlinearity, CCD, FOD, and ss seriously enhances the formation of MI sidebands, both in normal as well as anomalous dispersion regimes. For asymmetric cw, spatial MI is dependent on linear coupling term and quintic nonlinearity.

1. Introduction

In nonlinear fiber optics, much attention has been devoted to the investigations of MI in the framework of nonlinear Schrödinger equation. MI is a characteristic feature of a wide class of nonlinear dispersive systems. It is a fundamental nonlinear phenomenon [1–3] in which a weak perturbation imposed on a continuous wave (cw) state grows exponentially, which results in the break up of cw into a train of ultra-short pulses. MI has been studied since 1960s. This phenomenon arises due to interplay between nonlinearity and dispersion. MI plays an important role in many nonlinear phenomena such as cross phase modulation [2,4], four-wave mixing [5], supercontinuum generation [6], and Bragg grating [3,7]. MI can be classified into three main categories—spatial [8,9], temporal [10,11], and spatiotemporal [12,13]. The spatial MI occurs due to the interaction between the nonlinearity and diffraction which results into the break-up of homogeneous beam into numerous small filaments. The temporal MI occurs due to interplay between the group velocity dispersion (GVD) and nonlinearity and manifests itself as break-up of cw into a train of ultra-short pulses. In temporal MI the anomalous GVD plays the same role as is played by diffraction term in spatial MI. However in spatiotemporal MI all the three terms—nonlinearity, dispersion, and diffraction are nonzero and it occurs due to the simultaneous presence of spatial and temporal MI in nonlinear medium. Recently, the phenomenon of MI has been extensively explored in different areas such as negative refractive index materials [14,15], silicon photonic nanowires [16], and in single core fiber [17,18], however very less attention has been paid to the study of MI in twin-core fibers [19,20].

The planar dual-core waveguide is a waveguide that consists of two linearly coupled identical parallel cores. In a DWG optical power can be transferred between two cores periodically [21]. This phenomenon of periodic optical power transfer between the two cores along a DWG is widely used in many practical planar waveguide devices. Power transfer through DWG has two general continuous wave (cw) state:

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(1) The first state is symmetric/antisymmetric state in which the power through two cores of DWG are always equal. If the power in two cores are in phase then state is said to be symmetric state and if the power in two cores are out of phase then state is said to be asymmetric.
(2) The second state is antisymmetric state, where optical power in two cores are unequal.

The evolution of slowly varying envelope in DWG is governed by a set of coupled nonlinear Schrödinger equations (NLSE). The coupling coefficient for linear coupling between the two equations dictates the strength of the power transfer. The magnitude of coupling constant depends upon the design and operation condition of the optical fiber. The coupling coefficient dispersion (CCD) plays an important role in pulse distortion and it can also result into the pulse breakup and thus seriously affect nonlinear pulse switching [22,23].

Recently Li et al. [24] have investigated the effect of CCD on MI for twin-core fiber in the presence of GVD and cubic nonlinearity for symmetric/antisymmetric and asymmetric cw. In particular, the system of coupled NLSE does not include the diffraction, the FOD, and the quintic nonlinear term, and thus the characteristic of two other kinds of MI—spatial and spatio-temporal MI and the physics behind them are not disclosed. Also, recently, Kartashov et al. [25] studied stabilization of spatio-temporal solitons in Kerr media by dispersive coupling. Motivated by these work and others, we have investigated MI for DWG in the presence of CCD and other higher order effects such as quintic nonlinearity, self steepening, third order dispersion, and fourth order dispersion.

In particular, we have studied MI for both symmetric/antisymmetric and asymmetric cw. Three different types of MI —spatial, temporal, and spatiotemporal MI have been studied for symmetric/antisymmetric cw. We have investigated the variation of spatial MI with quintic nonlinearity for self-focusing and self-defocusing medium. For the temporal case we have studied the impact of various parameters such as coupling term, FOD, ss, and quintic nonlinearity on the MI gain in focusing as well as defocusing medium. For example, the spatial MI gain is directly proportional to the strength of quintic nonlinearity and coupling term, while the temporal MI gain crucially depends on strength of quintic nonlinearity, FOD and ss terms. Thirdly, the spatio-temporal MI can occur for focusing medium in the normal dispersion regime with an enhanced formation of sidebands, while for the defocusing nonlinearity and anomalous dispersion, there is a suppression of generation of MI sidebands. To sum up, we affirm that all these additional terms provide extra freedom to control the amplitude of the MI gain profile.

2. Model equations

In DWG, each core supports only a single mode. The evolution of the electric-field envelopes along the waveguide is described by a pair of generalized coupled nonlinear Schrödinger equations given by

$$\frac{\partial\Psi_1}{\partial\xi}=\frac{i}{2k_0}\frac{\partial^2\Psi_1}{\partial x^2}-\frac{i\beta_2}{2}\frac{\partial^2\Psi_1}{\partial\tau^2}+iC_{nl}(1+iC_s\frac{\partial}{\partial\tau})|\Psi_1|^2\Psi_1-\delta_3\frac{\partial^3\Psi_1}{\partial\tau^3}+i\delta_4\frac{\partial^4\Psi_1}{\partial\tau^4}+iC_q|\Psi_1|^4\Psi_1+\Gamma\psi_2+P\frac{\partial\Psi_2}{\partial\tau},$$
$$\frac{\partial\Psi_2}{\partial\xi}=\frac{i}{2k_0}\frac{\partial^2\Psi_2}{\partial x^2}-\frac{i\beta_2}{2}\frac{\partial^2\Psi_2}{\partial\tau^2}+iD_{nl}(1+iD_s\frac{\partial}{\partial\tau})|\Psi_2|^2\Psi_2-\delta_3\frac{\partial^3\Psi_2}{\partial\tau^3}+i\delta_4\frac{\partial^4\Psi_2}{\partial\tau^4}+iD_q|\Psi_2|^4\Psi_2+\Gamma\psi_1+P\frac{\partial\Psi_1}{\partial\tau}, \quad (1)$$

where ξ and τ are the propagation distance and time respectively. Ψ_1 and Ψ_2 are the slowly varying pulse envelopes in two cores, β_2, measures the GVD at the carrier frequency ($\beta_2<0$ for anomalous dispersion and $\beta_2>0$ for normal dispersion). C_{nl} and D_{nl} are the coefficients of cubic nonlinearity in two cores of the DWG. $C_{nl}C_s$ and $D_{nl}D_s$ self-steepening coefficients in two cores. δ_3,δ_4 are the coefficient of TOD and FOD respectively. C_q, D_q are the coefficients of quintic nonlinearity in two cores. Γ and P are linear coupling term coefficient and coupling coefficient dispersion (CCD) term respectively. Introducing the normalized units,

$$Z=\tfrac{\xi}{L_D},\quad t=\tfrac{\tau}{T_0},\quad U=\tfrac{\Psi_1}{\Psi_{01}},\quad V=\tfrac{\Psi_2}{\Psi_{02}},\quad X=\tfrac{x}{L_\perp},\quad u=N_1U,\quad v=N_2V,$$

where T_0 is the pulse width, $L_D=T_0^2/|\beta_2|$ is the dispersion length, and Ψ_{01} and Ψ_{02} are the initial amplitudes of the slowly varying envelope in two cores of the DWG. N_1 and N_2 may be termed the order of solitons and are defined as $N_1^2=L_D/L_{C_{nl}}$, $N_2^2=L_D/L_{M_{nl}}$, and we assume that $N_1=N_2=N$. We define nonlinear polarization length as $L_{C_{nl}}=1/C_{nl}\Psi_{01}^2$ and $L_{M_{nl}}=1/D_{nl}\Psi_{02}^2$. Let $g_1=\Psi_{01}^2C_{qnl}/(C_{nl}N^2)$ and $g_2=\Psi_{02}^2D_{qnl}/(D_{nl}N^2)$ and characteristic length $L_\perp=\sqrt{|L_D/k_0|}$ is also introduced. $\beta_3=\delta_3/T_0|\beta_2|,\beta_4=\delta_4/T_0^2|\beta_2|$ and $C_1=P\Psi_{02}/\Psi_{01}|\beta_2|$. Thus Eq. (1) can be transformed into the following form

$$\frac{\partial u}{\partial Z}=\frac{isgn(k_0)}{2}\frac{\partial^2u}{\partial x^2}-\frac{isgn(\beta_2)}{2}\frac{\partial^2u}{\partial t^2}+if(1+iS_1\frac{\partial}{\partial t})|u|^2u-\beta_3\frac{\partial^3u}{\partial t^3}+i\beta_4\frac{\partial^4u}{\partial t^4}+ig_1|u|^4u+\eta v+C_1\frac{\partial v}{\partial t},$$
$$\frac{\partial v}{\partial Z}=\frac{isgn(k_0)}{2}\frac{\partial^2v}{\partial x^2}-\frac{isgn(\beta_2)}{2}\frac{\partial^2v}{\partial t^2}+if(1+iS_2\frac{\partial}{\partial t})|v|^2v-\beta_3\frac{\partial^3v}{\partial t^3}+i\beta_4\frac{\partial^4v}{\partial t^4}+ig_2|v|^4v+\eta u+C_1\frac{\partial u}{\partial t}. \quad (2)$$

Here $S_1=|C_s|/T_0, S_2=|D_s|/T_0$ and $\eta=\Gamma\ T_0^2/|\beta_2|$.

3. Linear stability analysis

The continuous steady state solution of this equation is $u=a_0\exp(i\Omega_{0a}Z)$ and $v=b_0\exp(i\Omega_{0b}Z)$, where a_0 and b_0 are the normalized amplitude also Ω_{0a} and Ω_{0b} are corresponding nonlinear phase shift, which satisfies $\Omega_{0a}=fa_0^2+b_0^2+g_1a_0^4$ and $\Omega_{0b}=a_0^2+fb_0^2+g_2b_0^4$. For symmetric/antisymmetric cw $a_0=mb_0$, where $m=\pm1$ ($m=+1$ for symmetric and $m=-1$ for antisymmetric cw) and for asymmetric cw $a_0\neq b_0$. If continuous wave solution is slightly perturbed from the steady state,

$$u(X,Z,T)=[a_0+a(X,Z,T)]\exp(i\Omega_{0a}Z) \quad (3)$$

$$v(X,Z,T)=[b_0+b(X,Z,T)]\exp(i\Omega_{0b}Z) \quad (4)$$

where $a(X,Z,T)$ and $b(X,Z,T)$ are the perturbations such that $a,b\ll1$. Substituting Eq. (3) and Eq. (4) into Eq. (2) and linearizing in a and b, we obtain the following equations

$$\frac{\partial a}{\partial Z}=p\frac{i}{2}\frac{\partial^2a}{\partial X^2}-\frac{i\delta}{2}\frac{\partial^2a}{\partial T^2}+if\ a_0^2(a+a^*)-fS_1a_0^2(2\frac{\partial a}{\partial T}+\frac{\partial a^*}{\partial T})-\beta_3\frac{\partial^3a}{\partial T^3}+i\beta_4\frac{\partial^4a}{\partial T^4}+2ig_1b_0^4(a+a^*)+\eta b+C_1\frac{\partial b}{\partial T},$$
$$\frac{\partial b}{\partial Z}=p\frac{i}{2}\frac{\partial^2b}{\partial X^2}-\frac{i\delta}{2}\frac{\partial^2b}{\partial T^2}+ifb_0^2(b+b^*)-fS_2b_0^2(2\frac{\partial b}{\partial T}+\frac{\partial b^*}{\partial T})-\beta_3\frac{\partial^3b}{\partial T^3}+i\beta_4\frac{\partial^4b}{\partial T^4}+2ig_2a_0^4(b+b^*)+\eta a+C_1\frac{\partial a}{\partial T}. \quad (5)$$

Now substituting $a=a_1\exp(i(kZ-\Omega T+q_xX))+a_2\exp(i(kZ-\Omega T+q_xX))$ and similarly for b. Where $k,\Omega,q^2=q_x^2$ are the longitudinal wave number, frequency and transverse wave number of the