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Single-layer economic model predictive control for periodic operation

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ABSTRACT

In this paper we consider periodic optimal operation of constrained periodic linear systems. We propose an economic model predictive controller based on a single layer that unites dynamic real time optimization and control. The proposed controller guarantees closed-loop convergence to the optimal periodic trajectory that minimizes the average operation cost for a given economic criterion. A priori calculation of the optimal trajectory is not required and if the economic cost function is changed, recursive feasibility and convergence to the new periodic optimal trajectory is guaranteed. The results are demonstrated with two simulation examples, a four tank system, and a simplified model of a section of Barcelona's water distribution network.

1. Introduction

The main objective of a process control system is to guarantee closed-loop stability satisfying the process constraints while minimizing the operation cost. Model predictive control (MPC) has demonstrated to be a suitable control technique for optimal process operation thanks to its ability to yield high performance control systems capable of operating multivariable constrained systems without expert intervention for long periods of time [5,24].

Most industrial processes are typically operated at the equilibrium point that minimizes an economic cost function. Standard MPC approaches are based on a hierarchical structure in which the operation point is calculated by a real-time optimizer (RTO), and then a model predictive controller is designed to drive the system to this operation point [6,7]. In order to enhance the performance during the transient of the system, economic MPC has been proposed [3,22]. The main characteristic of this predictive controller is that the MPC cost function uses the economic cost function as stage cost function. This allows the controller to take into account the performance during the transient but brings stability issues. In [10–12], Lyapunov based model predictive control designs, which are capable of optimizing closed-loop performance with respect to general economic considerations for a broad class of nonlinear process systems, including systems subject to asynchronous and delayed measurements and uncertain variables; have been developed. The proposed techniques are based on two different modes of operation that guarantee that the closed-loop system is ultimately bounded in a small region containing the origin.

In [23], recent results on the stabilizing design of economic MPC are summarized. In [9] an stabilizing economic MPC without terminal constraint is presented. In [28] a single layer economic MPC has been proposed by integrating the RTO into the MPC as terminal cost function and the benefits of this controller has been practically validated.

It has been proved that steady state operation results to be optimal when the system is strictly dissipative with respect to the economic cost function [2]. However in certain cases, the optimal operation of the plant from an economic point of view is not to remain at a given steady state but to follow a non-steady trajectory, often periodical [17,14]. This is the case for instance when the system is subject to periodic disturbances (such as an exogenous periodic demand of water distribution networks or supply chains), fluctuating prices of the economic cost function (such as the electricity unitary cost), or time varying dynamics (such as batch nonlinear-process operation). In [13] it is reported that processes such as simulated moving bed (SMB) and pressure swing adsorption (PSA) present a better economic performance if they are operated following a periodic trajectory.

To deal with non steady periodic operation of a plant, the predictive control structure must be modified. One solution proposed

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in the literature is to follow a two layer approach in which the optimal periodic trajectory is calculated by a dynamic real time optimizer (DRTO), which takes into account a dynamic model of the plant; and based on this optimal periodic trajectory, a model predictive controller for tracking the optimal trajectory is applied, see for example [27]. If the model predictive controller is designed appropriately [24], asymptotic convergence of the closed-loop system to the optimal trajectory can be proved. In order to improve the economic performance during the transient, several authors propose to use economic MPC to track the optimal trajectory. In [2] an economic MPC that guarantees that the asymptotic average economic cost of the controlled system is no worse that the average economic cost of the optimal trajectory has been presented. In [14] an economic MPC for cyclic processes is presented. Lyapunov stability of the controlled plant is derived if the initial state is in a neighborhood of the optimal trajectory. In [13] stability and robustness of infinite horizon economic MPC is analyzed.

In general, all the above-mentioned control strategies require the calculation of the optimal periodic trajectory by the real time optimization layer for the given economic cost function. The economic cost function typically depends on exogenous parameters, such as unitary prices or expected demands that may be changed throughout the operation of the plant. When these parameters are changed, then the optimal trajectory must be recalculated and the predictive controllers should be re-designed to this new scenario by adapting the constraints and/or the cost function appropriately. The subsequent variation of the constraints of the optimization problem could lead to feasibility loss [20,8].

Motivated by these issues, in this paper we consider periodic optimal operation of constrained periodic linear systems. We propose an economic model predictive controller based on a single layer that unites dynamic real time optimization and control following the idea of [28]. The proposed controller guarantees closed-loop convergence to the optimal periodic trajectory that minimizes average operation cost for a given economic criterion. A priori calculation of this optimal trajectory is not required. In addition, if the economic cost function is changed, recursive feasibility and convergence to the new periodic optimal trajectory is guaranteed. The results are demonstrated with two simulation examples, a four tank system, and a simplified model of a section of Barcelona's water distribution network.

1.1. Notation

Bold letters are used to denote the sequence of T values of the signal, i.e. $\mathbf{z}_T = \{z(0), \ldots, z(T-1)\}$. $\mathbf{z}_T(k)$ denotes the sequence $\mathbf{z}_T(k) = \{z(k), \ldots, z(k+T-1)\}$; if the sequence depends on a parameter θ, then this is denoted as $\mathbf{z}_T(k;\theta) = \{z(k;\theta), \ldots, z(k+T-1;\theta)\}$. $\mathbb{I}_{[a,b]}$ denotes the set of integer numbers contained in the interval $[a, b]$, that is $\mathbb{I}_{[a,b]} = \{a, a+1, \ldots, b\}$. For a certain parametric optimization problem $\min_{z \in Z} F(z;\theta)$, the optimal solution is denoted as $z^o(\theta)$.

2. Problem formulation

In this work we focus on the following class of time-varying linear systems

$$x(k+1) = A(k)x(k) + B(k)u(k) + w(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $w(k) \in \mathbb{R}^n$ are the state, input and disturbance vectors of the system at time step k respectively. The evolution of the matrices $A(k)$ and $B(k)$ as well as the disturbance signal $w(k)$ are known.

The dynamic model can be considered as a time varying affine system denoted as

$$x(k+1) = f(k, x(k), u(k)) \tag{2}$$

with $f(k, x, u) = A(k)x + B(k)u + w(k)$.

The state and input must satisfy the following constraints

$$(x(k), u(k)) \in \mathcal{Z}(k) \subseteq \mathbb{R}^{n+m} \tag{3}$$

where $\mathcal{Z}(k)$ is a closed convex polyhedron that may vary in time. It is assumed that $\mathcal{Z}(k)$ is known and contains the origin in its interior.

The performance of the evolution of the plant is measured by an economic stage cost function $\ell(k, x, u, p)$ that depends on the current state and input of the plant, on the time and on an exogenous parameter p. The value of the parameter p may be changed during the operation of the plant and this variation is not known a priori. This function is assumed to be positive $\ell(k, x, u, p) \geq 0$ for all (k, x, u, p) and convex in (x, u) for all k and p.

Remark 1. No assumption is made on how the parameter p affects the cost function. It can be a numerical parameter that can be changed (as for instance unitary prices) or a switch between completely different and unrelated economic cost functions. The policy of these changes is not known and then the future evolution of p is uncertain. The variations of p may induce a dramatic variation on the optimal operation of the system. This is shown in the illustrative examples. A sudden change in p is equivalent to a set-point change for a standard MPC for regulation.

In this paper we focus on periodic operation of a closed-loop system with a fixed period T. The periodic behavior may be a consequence of the time-varying system dynamics, the exogenous disturbances, the constraints and/or the time varying economic stage cost function. Thus, these functions are considered to be periodic, as it is stated in the following assumption.

Assumption 1. The system is periodic and its period is T. That is, for all k, the following equations hold

$$\begin{aligned}
&A(k) = A(k+T)\\
&B(k) = B(k+T)\\
&w(k) = w(k+T)\\
&\mathcal{Z}(k) = \mathcal{Z}(k+T)\\
&\ell(k, x, u, p) = \ell(k+T, x, u, p), \quad \forall (x, u) \in \mathcal{Z}(k),\ \forall p
\end{aligned}$$

2.1. Economically optimal periodic operation

The main objective of the proposed control system is to operate the plant to achieve an optimal economic performance. The economic performance is measured with the average of the economic cost function of the closed-loop system trajectories, that is

$$L_\infty(0, x(0), \mathbf{u}_\infty(0), p) = \lim_{M \to \infty} \frac{1}{M} \sum_{j=0}^{M-1} \ell(j, x(j), u(j), p)$$

where $x(0)$ is the initial state and $\mathbf{u}_\infty(0)$ its corresponding closed-loop input trajectories.

The optimal trajectory in which the system can be operated $(\mathbf{x}^*_\infty, \mathbf{u}^*_\infty)$ is derived from the solution of the following optimization problem in which the initial state is a free variable

$$\min_{x(0), \mathbf{u}_\infty} \; L_\infty(0, x(0), \mathbf{u}_\infty, p) \tag{4a}$$

$$s.t.\ \ x(j+1) = f(j, x(j), u(j)) \tag{4b}$$

$$(x(j), u(j)) \in \mathcal{Z}_r(j), \quad \forall j \geq 0 \tag{4c}$$

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