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Handling uncertainty in economic nonlinear model predictive control: A comparative case study

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ABSTRACT

In the last years, the use of an economic cost function for model predictive control (MPC) has been widely discussed in the literature. The main motivation for this choice is that often the real goal of control is to maximize the profit or the efficiency of a certain system, rather than tracking a predefined set-point as done in the typical MPC approaches, which can be even counter-productive. Since the economic optimal operation of a system resulting from the application of an economic model predictive control approach drives the system to the constraints, the explicit consideration of the uncertainties becomes crucial in order to avoid constraint violations. Although robust MPC has been studied during the past years, little attention has yet been devoted to this topic in the context of economic nonlinear model predictive control, especially when analyzing the performance of the different MPC approaches. In this work, we present the use of multi-stage scenario-based nonlinear model predictive control as a promising strategy to deal with uncertainties in the context of economic NMPC. We make a comparison based on simulations of the advantages of the proposed approach with an open-loop NMPC controller in which no feedback is introduced in the prediction and with an NMPC controller which optimizes over affine control policies. The approach is efficiently implemented using CasADi, which makes it possible to achieve real-time computations for an industrial batch polymerization reactor model provided by BASF SE. Finally, a novel algorithm inspired by tube-based MPC is proposed in order to achieve a trade-off between the variability of the controlled system and the economic performance under uncertainty. Simulations results show that a closed-loop approach for robust NMPC increases the performance and that enforcing low variability under uncertainty of the controlled system might result in a big performance loss.

1. Introduction

Model predictive control (MPC) and its nonlinear counterpart (NMPC) have become popular control strategies both for the process industry and for control researchers. The main reason for their success is the ability of NMPC to deal with multivariable nonlinear systems with constraints. Usually, a model of the system is used to predict the behavior of the system and to compute a sequence of optimal control inputs by minimizing a certain tracking objective which is mapped to a cost function. However, the ultimate goal of process control is not to track as well as possible a certain set-point that has been previously generated, but to maximize the profit (or minimize the costs) of a process as it is discussed in [1]. For this reason, in the last years the traditional tracking cost function of the MPC scheme has frequently been replaced by a more general cost function that can represent the plant economics and different results have been reported in e.g. [2–4], and in [5].

One of the problems of this modification is that it is not possible to extend directly the existing tools and theory for the stability analysis of standard tracking MPC (see [6] for a review) and new approaches are required, as stated in [7]. The main reason for this is that the central argument on which most of the stability analysis relies, that is, the optimal cost of the MPC problem is a Lyapunov function of the closed-loop system, does not hold for economic MPC under the usual assumptions. Recently, several modifications of the classical approaches have been proposed to guarantee the stability of economic MPC schemes. A Lyapunov function to prove stability

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of economic NMPC based on a strong duality assumption has been proposed in [8]. This assumption is relaxed in [9] using dissipativity assumptions. These methods need the use of a terminal constraint or of a terminal set as in standard NMPC, which can be dropped by introducing further assumptions as shown in [10].

Interestingly, very little attention has been paid to another important implication of economic MPC: the economic operation of a system typically drives the system to its constraints. This, together with the fact that models of real systems are never perfect, makes the handling of the uncertainty in economic NMPC a crucial issue in order to guarantee constraint satisfaction in the presence of the uncertainty and the disturbances that act on the system.

The consideration of uncertainties in tracking model predictive control has been extensively studied in the literature under the heading of robust MPC, but very little has been reported for the case of economic MPC. One of the few works in this direction is [11], where input-to-state stability of economic NMPC for uncertain cyclic processes is studied. In [12], robustness with respect to changes in the constraint set is analyzed based on dissipativity assumptions and [13] discusses the case of probabilistic constraints for linear economic MPC under uncertainty. For the case of traditional tracking MPC approaches, there is a large amount of literature which often considers a min–max formulation of the problem as originally proposed in [14] and later in the context of MPC in [15]. The traditional min–max MPC approaches are usually called open-loop min–max because they do not consider the fact that at the next sampling time the optimization problem resulting from the MPC scheme will be solved again. Closed-loop approaches as described in [16] optimize over a sequence of control policies instead of over a sequence of control inputs, taking feedback into account and reducing strongly the conservativeness of the controller as illustrated in [17]. However, the optimization over control policies results in an infinite-dimensional optimization problem that is very difficult to solve in general. In order to solve this problem, different approximations have been proposed such as optimizing over state feedback policies [18] or affine policies parametrized on the uncertainty [19,20]. In both cases, a certain degree of suboptimality is introduced.

Another possibility for robust NMPC that has become popular in the last years is the use of tube-based methods, that were first proposed in the context of linear MPC in [21]. The main idea of tube-based MPC is to decouple the robust control problem into the nominal solution and a second controller (called ancillary controller) that tries to steer the real system that is affected by the uncertainty to a set near the nominal trajectory. The approach was extended to nonlinear systems in [22] and different modifications have been proposed in the last years leading to different computational complexities and degrees of conservativeness (see e.g. [23–25]).

The stability properties of the mentioned strategies are always discussed in the literature for the case of tracking MPC. However, with the exception of [26], which proposes a method to estimate the suboptimality introduced by the choice of affine policies in the case of tracking MPC of linear systems, very little attention has been devoted to the analysis of the performance or conservativeness introduced by the different robust approaches, especially in the case of economic nonlinear model predictive control. For this reason, this paper focuses on analyzing the performance of different methods using robust economic NMPC for an industrially relevant batch polymerization reactor example provided by BASF SE. The theoretical stability analysis of the controllers is out of the scope of this paper and is not discussed.

In particular, we use the framework of multi-stage NMPC and we perform a comparison based on simulations with other typical robust MPC approaches by solving a challenging benchmark problem of an industrial polymerization reactor under large model uncertainty. Multi-stage NMPC uses ideas of closed-loop robust MPC [17] and is based on the representation of the uncertainty by a scenario tree. The multi-stage formulation represents the real-time decision problem that arises from the receding horizon strategy of MPC correctly because the future control inputs can depend on the realization of the uncertainty that will have been observed at the respective decision point, thus reducing the conservativeness of the approach considerably. Multi-stage NMPC has demonstrated a good potential for solving robust NMPC problems, as was demonstrated in [27,28]. The main drawback of the approach is that the size of the resulting optimization problem grows rapidly with the prediction horizon and with the number of uncertainties considered. This paper extends the results presented in [29] in several ways. The benchmark problem has been extended to consider an important safety constraint that modifies the optimal solution of the problem and we consider larger uncertainties. We compare the multi-stage approach with an open-loop robust NMPC approach and with robust NMPC using affine policies. We also present another novel contribution: a modification of the multi-stage NMPC approach inspired by tube-based MPC ideas in order to specify a trade-off between the variability of the trajectories for the different scenarios and the average economic performance. All optimization problems are solved in a very efficient way using CasADi, which makes it possible to solve the resulting Nonlinear Programming (NLP) problems with exact first and second order derivative information with a low implementation effort, significantly reducing the computation times reported in [28].

The remainder of the paper is structured as follows. Multi-stage NMPC is explained in Section 2 together with the implementations of open-loop robust NMPC and robust NMPC based on affine control policies that are applied in this work. Section 3 presents the case study considered in this paper: an industrial batch polymerization reactor provided by BASF SE. The numerical approach for the solutions of the resulting optimization problems using CasADi is presented in Section 3. A comparison which shows the benefits of multi-stage NMPC over traditional methods to achieve robustness (i.e. tracking a conservative-set point with a conservative choice of the uncertain parameters) is presented in Section 4. A comparison of multi-stage NMPC with other robust NMPC approaches is made in Section 5 and the modification of the multi-stage approach to reduce the variability of the system for the different values of the uncertainty is presented in Section 6. The main conclusions of the paper and future directions of work are stated in Section 7.

2. Robust economic nonlinear model predictive control

In this section, we describe several approaches to deal with uncertainty in model predictive control and we formulate all of them within the framework of multi-stage NMPC to enable a fair comparison for the proposed case study. First, we revisit the multi-stage NMPC approach and then the framework is extended to incorporate an open-loop approach in which a sequence of control inputs is calculated to satisfy the constraints for all the scenarios in the tree. Finally, the use of affine feedback control policies as a way to introduce feedback in the predictions of the NMPC scheme as described e.g. in [20] is shortly presented in the context of multi-stage NMPC.

2.1. Multi-stage nonlinear model predictive control

Multi-stage NMPC is a robust NMPC approach that is based on describing the evolution of the uncertainty by a scenario tree as depicted in Fig. 1. The tree structure visualizes that future control inputs can depend on the previous values of the uncertainty if full state measurement or perfect estimation is assumed, and thus can