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$\mathcal{H}_\infty$ model predictive control for discrete-time switched linear systems with application to drinking water supply network

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ABSTRACT

This paper investigates the problem of a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ model predictive control (MPC) for a class of discrete-time switched linear systems in the presence of constraints on the states and inputs and with norm bounded disturbances. The objective is to minimize the upper bound of an infinite horizon cost function subject to a terminal inequality by using induced $\mathcal{L}_2$-norm bound. This work is an extension of the MPC approach proposed in Kothare et al. (1996) to switched systems under arbitrary switching law and with additive disturbances. The switched structure of the system is taken into account and by using some mathematical tools, the predictive controller design problem is turned into a Linear Matrix Inequalities (LMIs) feasibility problem. Finally, the main results of this method are applied to a drinking water supply network (DWSN). More precisely, a MPC strategy to regulate the water storage in each tank, in the presence of physical structural modifications, is proposed. Simulation results are proposed to show the good convergence properties of the control.

1. Introduction

Over the past two decades, there has been significant research activities on hybrid systems. This category of systems has been reported in different fields (biology, electronics, chemical process, etc.) and the diversity of their behavior has been the subject of different papers [2–4]. Hybrid systems can be modeled using different formalisms, such as mixed logical dynamical (MLD) systems [5,6]; piecewise affine (PWA) systems [2,7–9]; and switched systems [10,11].

The control of switched systems is a methodology mostly used for systems having several operational modes. In each mode the evolution of the continuous state of the system is described by its own differential equation. The system switches among the various modes when a particular event occurs. These events can have different origins: they can be caused by variables crossing specific thresholds (state events) or by external inputs (input events). One challenging issue, when dealing with switched systems, is how to derive stabilizing switching feedback control laws. In this context, it is generally admitted that a necessary and sufficient condition for two linear subsystems to be quadratically stable, is the existence of a stable convex combination of the two subsystems [12,13]. A generalization to more than two linear subsystems was suggested by [14] using a “min-projection strategy”. However, for switched linear systems with more than two modes, the existence of a stable convex combination becomes only sufficient (see for instance [15]). Another approach, purely numeric, for robust stabilization of polytopic uncertain switched systems was proposed in [16]. In that paper, a quadratic stabilizing switching law was designed and the solution was formulated in terms of a LMI. All these methods use common quadratic Lyapunov functions to guarantee the stability. To reduce the conservatism of such a choice, the scientific community has been searching for another type of Lyapunov functions, called multiple Lyapunov functions (MLFs). This type of functions has been used for linear systems [17] nonlinear systems [18] and with the concept of dwell time [19,20]. Though many results on control synthesis of switched systems with or without uncertainties exist, only few results handle the context of the MPC methodology, as can be seen in [21–25].

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The MPC task is defined as an optimization problem under constraints (on the controlled variables and/or the state variables of the model) and is generally based on a cost function that has to be minimized on a given prediction horizon. Consequently, an optimal input sequence is obtained. Then, the first component of this sequence is applied after which the procedure is repeated again [1,26–28]. The cost function is generally defined using two forms: The first one is quadratic for which the MPC optimization problem leads to a Mixed Integer Quadratic Programming (MIQP) problem. In this case, an interesting work developed in [5] imposes a terminal equality constraint in order to guarantee the attractivity, instead of the Lyapunov stability. Nevertheless, this method requires the controllability of the PWA system rather than the stabilizability condition usually used. Besides, this method increases the computational burden resulting from using a longer prediction horizon. This approach was recently extended to a priori stabilization conditions that guarantee the stability of a class of piecewise affine systems [8,29,30]. The second form is based on either the 1-norm or the ∞-norm in which the MPC optimization problem leads to a Mixed Integer Linear Programming (MILP) approach. For instance, an a priori heuristic based MPC ∞-norm for guaranteeing the attractivity of PWA systems is addressed in [31]. In [32], the authors propose a posteriori stability check approach. More precisely, the PWA solution of the MPC optimization problem is computed explicitly and the stability of the closed-loop system is analyzed using piecewise quadratic Lyapunov functions. In the switched systems framework, the authors of [21] developed a Lyapunov-based predictive control method for systems with prescribed scheduled mode transitions. The stability of the switched closed-loop system is guaranteed if certain transition constraints imposed on the MPC design problem are satisfied. This result was extended in [33], where a robust MPC for the linear discrete-time case was proposed and the stability of the closed-loop was ensured by optimizing the switching signal which was considered as a design parameter. In [34], the authors used state dependent weighting matrices in the cost function. This method reduces the use of the actuators when their action is not required and forces the state trajectories to avoid critical regions in the system state space.

In this paper, a robust MPC design methodology for a class of discrete-time switched linear systems is investigated. In order to position our work with respect to the existing results, we summarize the main contribution as follows. Firstly, the MPC approach proposed in [1,35] is extended to switched systems under arbitrary switching law and with additive disturbances. Taking into account the switching character of the system, a state feedback controller for each subsystem is designed. In addition, a multiple-Lyapunov quadratic function to prove the asymptotic stability in closed-loop is considered. Secondly, to guarantee both the performance and robustness in the presence of disturbances, a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control is introduced. More precisely, the $\mathcal{H}_2$ norm of a quadratic cost function subject to an $\mathcal{H}_\infty$ norm constraint of another closed-loop transfer function is minimized under input and state constraints. This was already done in [35] though not for a switched system. Finally, less conservative and tractable LMIs stability conditions are provided by the use of some mathematical tools as Schur complement lemma, congruence transformation and the known extra instrumental variable used in [36,37]. As an application, we consider the problem of control of water volume in a drinking water supply network (DWSN) system. Generally, the DWSN is affected by structural modifications (e.g. multidirectional valves) which might change the nominal model of the network. For this reason, we model the network as a switched system in presence of these structural modifications (SMs). Then, we propose a strategy of control to regulate the motorized valves and water volume of each tank in order to ensure the daily drinking water consummation for the consumers.

This paper is organized as follows. Section 2 presents sufficient LMIs conditions for the asymptotic stability of the MPC controller. Furthermore, at the end of this section we present two numerical examples to show the validity and applicability of the developed theoretical result. In Section 3, the obtained results are applied in order to control the storage of water in a DWSN. In the last section, a conclusion is formulated.

Notations. The following notations will be used throughout this paper.

In a matrix, the notation $(\star)$ is used for the blocks induced by symmetry. $x(k)$ is the value of vector x at time k, predicted at time k. $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $\mathcal{L}_2^n[0,\infty)$ is the space of square summable infinite sequences and for $x=\{x(k)\}\in\mathcal{L}_2^n[0,\infty)$, its norm is given by $\|x\|_{\mathcal{L}_2^n}=\left(\sum_0^\infty\|x_k\|^2\right)^{1/2}$. In addition, for a positive matrix $S>0$, $\|x\|_S^2=x^TSx$.

2. Result on robust model predictive control for switched linear systems

2.1. Problem formulation

Let us consider the discrete-time system

$$x(k+1)=A_{\sigma(k)}x(k)+B_{\sigma(k)}u(k)+W_{\sigma(k)}w(k) \tag{1}$$

$$z(k)=\begin{bmatrix} C^z_{\sigma(k)}x(k) \\ D^z_{\sigma(k)}u(k)\end{bmatrix} \tag{2}$$

where $x(k)\in\mathbb{R}^n$, $u(k)\in\mathbb{R}^m$ and $z(k)\in\mathbb{R}^q$ are the state, control input and controlled output respectively. $w(k)\in\mathbb{R}^r$ is a disturbance in the defined set $\mathbb{W}=\{w\in\mathbb{R}^r \mid \|w\|_{\mathcal{L}_2^r}\le w_{\max}\}$. $\sigma(k)$ is a switching rule which takes its values in the finite set $\mathcal{I}=\{1,\ldots,s\}$, $s>1$ design the number of discrete modes. This means that the switched system is described by the set of modes

$$\{(A_i,B_i,W_i,C_i^z,D_i^z)|i\in\mathcal{I}\}.$$

and that the evolution of $\sigma(k)$ gives the switching sequence between these modes. In addition, the constraints for the state $x(k+1)$ and the control $u(k)$ are described as

$$\mathbb{U}=\{u(k): -u^d_{\lim}\le u^d(k)\le u^d_{\lim},\ d=1,\ldots,m\} \tag{3}$$

$$\mathbb{X}=\{x(k+1): -g^d_{\lim}\le x^d(k+1)\le g^d_{\lim},\ d=1,\ldots,n\} \tag{4}$$

for $k\ge 0$. $u_{\lim}$ and $g_{\lim}$ are vectors of suitable dimensions. $\mathbb{U}\subset\mathbb{R}^m$ and $\mathbb{X}\subset\mathbb{R}^n$ are closed subsets containing the origin in their interior. In the following, we make the following assumption: