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Optimal control of a large thermic process

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ABSTRACT

The studied problem concerns the optimal regulation with a fixed horizon of two vertical ovens having respectively three or twelve heating areas. The model of the thermal process is described thanks to a linear state equation with a quadratic criterion to minimize. Furthermore, the terminal state is fixed and the state is subjected to some constraints. We propose and analyze the relaxation method coupled with the shooting method to solve this problem. The convergence of the numerical procedure is presented. Numerical experiments are done and validate our approach.

1. Introduction

In the present study, we consider a numerical method for the determination of the control of a large physical system when the state is submitted to some constraints and moreover, when the final value of the state is fixed. Then, we apply the proposed method for the regulation of two vertical ovens decomposed respectively into three or twelve heating zones. This system presents great internal couplings, due to the natural convection in the chimney and to the thermal conduction; the objective is to maintain, in spite of perturbations, a prescribed distribution of temperature on a vertical object placed in the chimney. The observations are performed at n equidistant points. In this work, we are interested by the minimization of a linear quadratic optimal control problems with a fixed terminal time. Thus the studied problem is very complex. Nevertheless, we distinguish, under the same formalism, two separate cases of optimal control problems: the case without constraint and the case with constraints on the state; in the second case, we use the concept of sub-differentiability [1,3] allowing to take into account, if necessary, the projection on the convex set of constraints. Thus, in order to solve this problem, we first reformulate the necessary optimality conditions. So, the constraints applied to the state are taken into account by considering the sub-differential mapping of the indicator function of the convex set which models the constraints. Thus, we obtain a multi-valued formulation of the necessary optimality conditions derived from the Pontryagin's minimum principle; this multi-valued formulation comes from the perturbation of the system to control by the sub-differential mapping of the indicator function of the convex set. Then, we obtain a system in which, the differential equation describing the state is equipped with an initial and a final conditions. In order to determine the initial condition of the costate, we use in this study, the shooting method (see [10] for a general presentation and, in a more particular context [7] for the use of such method in the case where no constraint is applied on the state and where the command is bang-bang), coupled with the relaxation method (see [2,5,9]). Due to the properties of monotonicity of on one hand the sub-differential mapping of the indicator function and on the other hand under appropriate assumptions of the derivatives of the state and of the costate with respect to the time, we can prove the convergence of the numerical relaxation algorithm coupled with the shooting method used in order to satisfy the second constraint concerning the final value of the state. This theoretical study constitutes the main result of the presented work. Moreover, the application of the proposed method to the regulation of a large thermic process, constitutes an additional contribution. Indeed it is interesting and meaning to show some results concerning a real application with a large state equation for the regulation of two

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ovens. In particular, the state of one studied thermic process is described with 24 constrained ordinary differential equations and we have to solve an algebro-differential system with 60 equations.

The application of such challenges in controlling vertical ovens is classical in many fields (see for example references [2,5]) in which we have considered the same model of problem when the state is not submitted to constraints and when we have a free final value of the state. In this last case the problem to solve is simpler and it is not necessary to use the shooting method; thus the computation procedure is different and the present study is more difficult.

This paper is organized as follows. In Section 2, we define the problem in both cases with and without constraints on the state and we present the shooting method. Section 3 is devoted to the description of the relaxation method coupled with the shooting one. The convergence of the proposed method is presented in Section 4, while Section 5 contains the results of the numerical experiments.

2. Regulation of a thermal process

2.1. Case without constraints

We consider a physical system composed of a vertical oven, in the chimney of which is placed a bar (Fig. 1). The goal is to bring the temperature z identified in n points of the bar at a desired temperature z_d, in a finite time T. In the sequel, $x(t) \in \mathbb{R}^n$ represents the temperature of the chimney in n points. $u(t) \in \mathbb{R}^m$ is the vector of control representing the intensity of currents applied to the n heating zones. We shall therefore have to determine the control u, so that, at the final time T, the temperature z of the bar is uniformly equal to z_d, with a criterion to be minimized which is specified below. Let $y = (z_1, x_1, \ldots, z_i, x_i, \ldots, z_n, x_n)$ be a state vector of the system; then the mathematical model is obtained by linearization around a working point of the heat equation and is represented as follows

$$\begin{cases} \dot{y}(t) = Ay(t) + Bu(t), \quad t \in [0, T], \\ y(0) = y_0, \quad y(T) = y_f, \\ u(t) \in U, \end{cases} \tag{1}$$

where $y(0) = y_0$ is the initial state, $y(T) = y_f$ is the final state. U is the set of admissible controls, which is an open set. $A \in \mathbb{R}^{2n\times 2n}$ and $B \in \mathbb{R}^{2n\times m}$ are two given constant matrices. The observation of the system is constituted by a part of the vector x namely the vector z; let

$$z = Cy, \tag{2}$$

where $C \in \mathbb{R}^{n\times 2n}$ is the observation matrix. The problem is completed by the minimization of a functional $J(u)$:

$$J(u) = \frac{1}{2}\int_0^T \left[\frac{\alpha}{\|z_d\|_2^2}\|z - z_d\|_2^2 + \frac{\beta}{\|u_d\|_2^2}\|u - u_d\|_2^2 \right] dt, \tag{3}$$

where u_d is the control which leads asymptotically to the prescribed temperature z_d; it can be shown obviously that

$$u_d = -(CA^{-1}B)^{-1} z_d.$$

The two coefficients α and β indicate the comparative weights given to the two components of the cost function, i.e., the values of α and β are chosen in order to provide respectively more weight to precision or energy expenditure.

Thus, the problem is to find an admissible optimal control law that makes it possible the transfer of the system from an initial state y_0 to the fixed final state y_f and which minimizes the cost functional (3).

The Hamiltonian of the system (1)–(3) is given by:

$$H(y, p, u, t) = \frac{1}{2}\left[\frac{\alpha}{\|z_d\|_2^2}\|z - z_d\|_2^2 + \frac{\beta}{\|u_d\|_2^2}\|u - u_d\|_2^2 \right] + p^t[A \quad y + B \quad u],$$

where p is the costate vector. Now, we have to find the function $\hat{u}$ which minimizes the Hamiltonian, i.e., such that

$$H(\hat{y}, \hat{p}, \hat{u}) \le H(y, p, u); \quad \forall u \in U, \quad \forall t \in [0, T].$$

In the case without constraint on the state, the optimality conditions can be written as follows:

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{\partial H}{\partial p} = Ay + Bu; \quad y(0) = y_0, \quad y(T) = y_f, \quad \forall t \in [0, T], \\ -\dfrac{dp}{dt} = \dfrac{\partial H}{\partial y} = A^t p + C^t Cy - C^T z_d, \quad p(0) \ \text{to be determined}, \\ \dfrac{\partial H}{\partial u} = 0 = B^t p + k(u - u_d), \end{cases} \tag{4}$$

where

$$k = \frac{\|z_d\|^2}{\|u_d\|^2}\frac{\beta}{\alpha}.$$

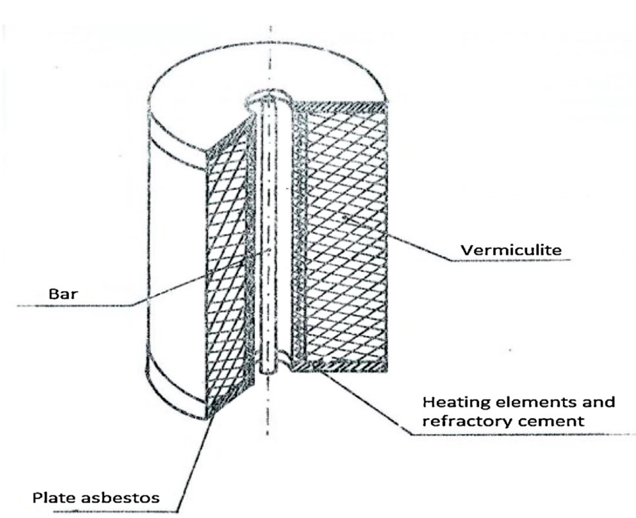

Fig. 1. Representation of the oven.

2.2. Case with constraints on the state

In the case where the state is submitted to some bounded constraints denoted respectively by y^{min} and y^{max}, let Y_{ad} be the convex set of admissible trajectories. In the sequel, we will reformulate the necessary conditions of optimality. Thus, we will use the notion of sub-differential to obtain the optimality conditions. Let us first recall few mathematical notions.

Definition 1. Let E be a vectorial space. Let χ, be a proper convex function on E and a point $\mu \in E$, we denote by $\partial\chi(\mu)$ the set of all $\mu' \in E'$ such that

$$\chi(\upsilon) \ge \chi(\mu) + \langle \upsilon - \mu, \mu' \rangle, \ \text{for every}\ \upsilon \in E, \tag{5}$$

where $\langle , \rangle$ denotes the duality product and E' is the dual topological space associated to the vectorial space E. Such an element μ' is called the sub-gradient of χ at μ, and $\partial\chi(\mu)$ is called the sub-differential of χ at μ.